

Econ 702 - Week 2

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1 Solow growth Model

1.1 Review

- Firm

- Production function

$$Y_t = A_t F(K_t, N_t)$$

- Firms rent capital at a rate r_t and pay workers a wage of w_t . Those variables are exogenous to the firm. It chooses K_t and N_t to maximize

$$\Pi_t = A_t F(K_t, N_t) - r_t K_t - w_t N_t$$

- The FOCs are

$$w_t = A_t F_N(K_t, N_t), \quad r_t = A_t F_K(K_t, N_t)$$

- Household

- The household owns capital stock K_t , which it can rent to firms.
- Each household is also endowed with N_t unit of labor which it supplies to firm.
- It is assumed that a constant fraction of income is saved.

$$I_t = sY_t, \quad C_t = (1 - s)Y_t$$

- The equations of Solow model and their corresponding per worker terms are

- $Y_t = C_t + I_t$
- $Y_t = A_t F(K_t, N_t)$
- $K_{t+1} = I_t + (1 - \delta)K_t$
- $I_t = sY_t$
- $w_t = A_t F_N(K_t, N_t)$
- $r_t = A_t F_K(K_t, N_t)$

1.2 Exercise: Simple two country model - Allocation Puzzle

Suppose there is only one period and there are two countries A and B. In country A, there is one representative household who owns capital stock $K_A = \bar{K} > 0$ and one unit of labor $N_A = 1$. There is one representative firm who produces by renting the capital with rental rate r_A and by hiring labor with wage w_A in country A. The production function is

$$Y_A = A_A(\hat{K}_A)^\alpha(\hat{N}_A)^{1-\alpha}, \quad \alpha \in (0, 1)$$

Country B is symmetric except

$$A_A < A_B.$$

1. Write down the firm's profit in country A and country B. Derive the FOCs with respect to the capital choice \hat{K}_A and \hat{K}_B .

Answer:

The profits are

$$\Pi_A = A_A(\hat{K}_A)^\alpha(\hat{N}_A)^{1-\alpha} - r_A\hat{K}_A - w_A\hat{N}_A$$

$$\Pi_B = A_B(\hat{K}_B)^\alpha(\hat{N}_B)^{1-\alpha} - r_B\hat{K}_B - w_B\hat{N}_B$$

Then we can derive the FOCs as follows

$$\alpha A_A(\hat{K}_A)^{\alpha-1}(\hat{N}_A)^{1-\alpha} = r_A$$

$$\alpha A_B(\hat{K}_B)^{\alpha-1}(\hat{N}_B)^{1-\alpha} = r_B$$

2. Suppose there is no international capital flow, i.e. the firm in country A only rents the capital from country A household and the firm in country B only rents the capital from country B household. Calculate the market the rental rate of capital r_A^* and r_B^* under no international capital flow.

Answer:

Since there is no international capital flow and there is one unit of labor supply in each country, capital and labor markets clear with

$$\hat{K}_A = \bar{K}, \quad \hat{K}_B = \bar{K}, \quad \hat{N}_A = 1, \quad \hat{N}_B = 1$$

Substituting those into the firms' FOC gives

$$r_A^* = \alpha A_A(\bar{K})^{\alpha-1}, \quad r_B^* = \alpha A_B(\bar{K})^{\alpha-1}$$

3. Suppose there is a international capital renting market with rental rate r^* with $r_A^* < r^* < r_B^*$. Then which country rents from the rest of the world? Which country rents to the rest of the world?

Answer:

Since the world rental rate is given by r^ and labor supply is still 1 in each country, a firm in each country chooses to rent the capital stock satisfying*

$$\alpha A_A(\hat{K}_A)^{\alpha-1} = r^*$$

$$\alpha A_B(\hat{K}_B)^{\alpha-1} = r^*$$

For country A, since

$$\alpha A_A(\bar{K})^{\alpha-1} = r_A^* < r^* = \alpha A_A(\hat{K}_A)^{\alpha-1}$$

The capital stock held by household \bar{K} is larger than the capital stock rent by firm \hat{K}_A . Thus, country A will rent to the rest of the world.

For country B, since

$$\alpha A_B(\bar{K})^{\alpha-1} = r_B^* > r^* = \alpha A_B(\hat{K}_B)^{\alpha-1}$$

The capital stock held by household \bar{K} is smaller than the capital stock rent by firm \hat{K}_B . Thus, country B will rent from the rest of the world.

4. Gourinchas and Jeanne (2013) show that cross country correlation between the capital inflow and productivity growth is negative among developing countries. That is, countries with higher productivity and investment opportunity experience capital outflow. Is this finding consistent with the model prediction?

Answer:

In the model, more productive country (Country B) rents from the rest of the world (Capital inflow) and less productive country (Country a) rents to the rest of the world (Capital outflow). Therefore, the model prediction is at odds with the empirical pattern.

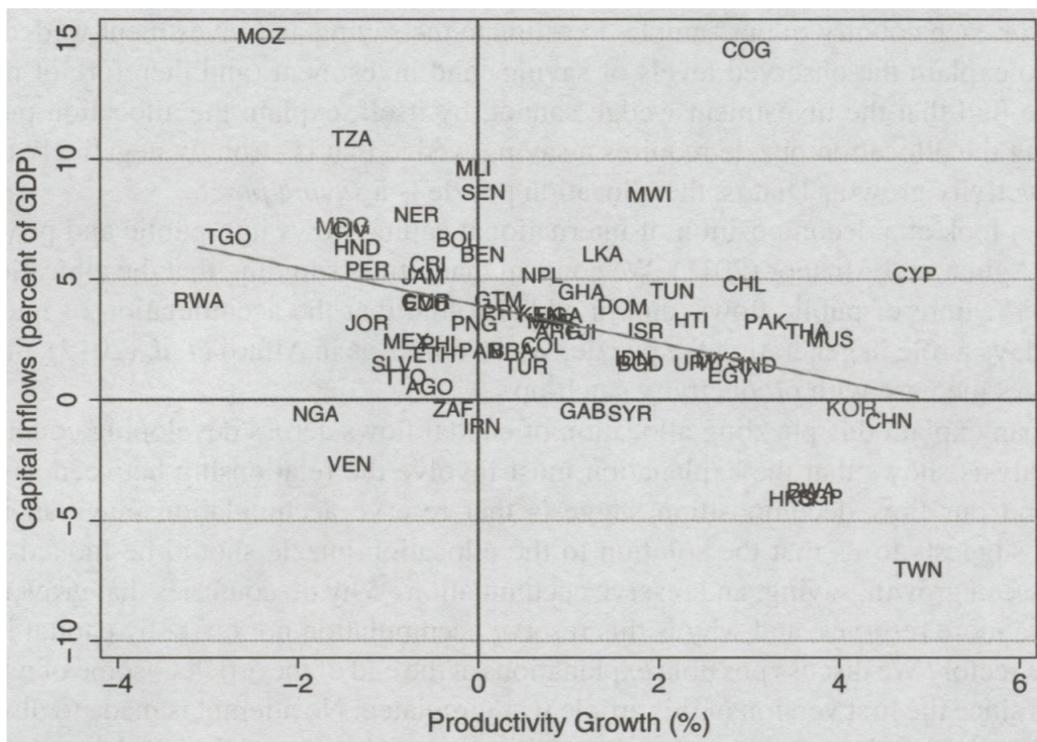


FIGURE 1

Average productivity growth and average capital inflows between 1980 and 2000. 68 non-OECD countries

(Gourinchas and Jeanne (2013))