Econ 702 - Week 4

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1 Review

1.1 Human Capital Model

- The depreciation rate δ is zero.
- Consumption:
$$\frac{C_{t+1}}{C_t} = \beta R_{t+1}, \quad where \quad R_{t+1} = \alpha A K_{t+1}^{\alpha-1} + 1$$
- Capital accumulation:
$$K_{t+1} - K_t = AK_t^{\alpha} - C_t$$
- Dynamics: Phase Diagram (in the exercise).
- Steady state:
$$K^* = \left(\frac{\beta\alpha A}{1-\beta}\right)^{\frac{1}{1-\alpha}}, \quad C^* = AK^{*\alpha}$$

1.2 Augmented Solow Model

- Aggregate Production Function
$$Y_t = AF(K_t, Z_t N_t)$$

  Here, Z_t is Labor-Augmented Productivity

  : **Discussion –**

  i) Why do we need this? ii) What is the real life example of this?

- Key equation for the evolution of capital stock is
$$\hat{k}_{t+1} = \frac{1}{(1+z)(1+n)}\left[sAf(\hat{k}_t) + (1-\delta)\hat{k}_t\right]$$
- For the Cobb-Douglas Case, the steady state value of $\hat{k}^*$ is
$$\hat{k}^* = \left(\frac{sA}{n+z+\delta}\right)^{\frac{1}{1-\alpha}}$$

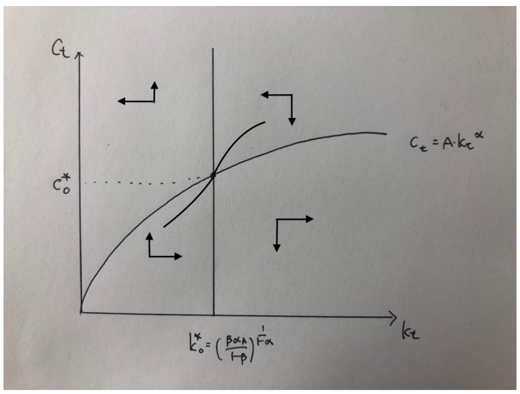
2 Exercise

2.1 Human Capital

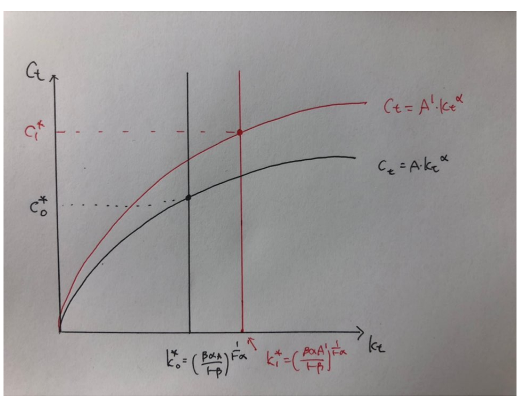
Recall the Human capital model we learned in the class. There are two important equation determining the dynamics of consumption and capital stock:

$$\frac{C_{t+1}}{C_t} = \beta\left(\alpha A K_{t+1}^{\alpha-1} + 1\right) \quad \textit{and} \quad K_{t+1} - K_t = AK_t^{\alpha} - C_t$$

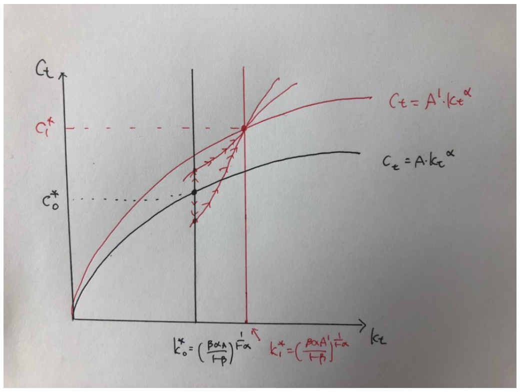
1. Draw a phase diagram to illustrate the steady state.
   Answer:



2. Suppose there is a permanent increase in productivity, i.e. $A' > A$. Compare the initial and the new steady state in a phase diagram. Then show the transitional dynamics of $\{C_t, K_t\}$ from the initial steady state to the new one.
   Answer:



As we can see in the phase diagram, new steady state consumption and capital are higher than before.



Also, depending on how patient the household is, the initial consumption could go up or down upon the change of A but it converges to the new steady state eventually followng the saddle path. Upon the change of A, the capital stock doesn't change but converges to the new steady state in the following periods.

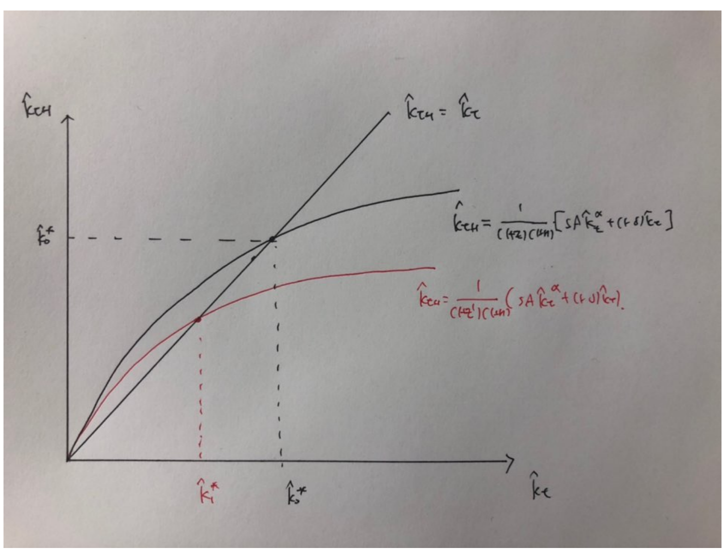
2.2 Augmented Solow Model

Suppose that you have a standard Solow model with a Cobb-Douglas production function and both labor augmenting productivity growth and population growth.

1. Write down the central equation for capital accumulation.
   Answer:

$$\hat{k}_{t+1} = \frac{1}{(1+z)(1+n)} \left[sA\hat{k}_t^\alpha + (1-\delta)\hat{k}_t \right]$$

2. Suppose that the economy initially sits in a steady state. Suppose that at time t there is a surprise increase in z that is expected to last forever. Use the main diagram to show how this will impact the steady state capital stock per efficiency unit of labor.
   Answer:



Let $\hat{k}_0^*$ be the steady state capital stock per efficiency unit of labor before the change and $\hat{k}_1^*$ be the level after the change. Then as we can see in the diagram, the new steady state level of $\hat{k}_1^*$ is less than the original one $\hat{k}_0^*$.

3. Plot out a diagram showing how the capital stock per efficiency unit of labor ought to react dynamically to the surprise increase in z.
Answer: