

Econ 702 - Week 4

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1 Review

1.1 Human Capital Model

- The depreciation rate δ is zero.
- Consumption:

$$\frac{C_{t+1}}{C_t} = \beta R_{t+1}, \quad \text{where } R_{t+1} = \alpha A K_{t+1}^{\alpha-1} + 1$$

- Capital accumulation:

$$K_{t+1} - K_t = AK_t^\alpha - C_t$$

- Dynamics: Phase Diagram (in the exercise).
- Steady state:

$$K^* = \left(\frac{\beta \alpha A}{1 - \beta} \right)^{\frac{1}{1-\alpha}}, \quad C^* = AK^{*\alpha}$$

1.2 Augmented Solow Model

- Aggregate Production Function

$$Y_t = AF(K_t, Z_t N_t)$$

Here, Z_t is Labor-Augmented Productivity

: **Discussion** –

i) Why do we need this? ii) What is the real life example of this?

- Key equation for the evolution of capital stock is

$$\hat{k}_{t+1} = \frac{1}{(1+z)(1+n)} \left[sAf(\hat{k}_t) + (1-\delta)\hat{k}_t \right]$$

- For the Cobb-Douglas Case, the steady state value of \hat{k}^* is

$$\hat{k}^* = \left(\frac{sA}{n+z+\delta} \right)^{\frac{1}{1-\alpha}}$$

2 Exercise

2.1 Human Capital

Recall the Human capital model we learned in the class. There are two important equation determining the dynamics of consumption and capital stock:

$$\frac{C_{t+1}}{C_t} = \beta \left(\alpha A K_{t+1}^{\alpha-1} + 1 \right) \quad \text{and} \quad K_{t+1} - K_t = A K_t^\alpha - C_t$$

1. Draw a phase diagram to illustrate the steady state.
2. Suppose there is a permanent increase in productivity, i.e. $A' > A$. Compare the initial and the new steady state in a phase diagram. Then show the transitional dynamics of $\{C_t, K_t\}$ from the initial steady state to the new one.

2.2 Augmented Solow Model

Suppose that you have a standard Solow model with a Cobb-Douglas production function and both labor augmenting productivity growth and population growth.

1. Write down the central equation for capital accumulation.
2. Suppose that the economy initially sits in a steady state. Suppose that at time t there is a surprise increase in z that is expected to last forever. Use the main diagram to show how this will impact the steady state capital stock per efficiency unit of labor.
3. Plot out a diagram showing how the capital stock per efficiency unit of labor ought to react dynamically to the surprise increase in z .