Econ 702 - Week 3

Professor: Charles Engel, TA: Junhyong Kim & Saerang Song

1 Solow Growth Model

1.1 Review

- Equations in per worker terms are
  - $y_t = Af(k_t)$
  - $c_t = (1-s)Af(k_t)$
  - $i_t = sAf(k_t)$
  - $k_{t+1} = (1-\delta)k_t + sAf(k_t)$
  - $R_t = Af'(k_t)$
  - $w_t = Af(k_t) - k_t Af'(k_t)$
- Steady State
  - An economy will eventually converge to the steady state regardless of any initial starting point. At the steady state the capital per worker does not change over time.
  - The steady state level of capital per worker (k^*) is determined by the following equation

$$k^* = sAf(k^*) + (1-\delta)k^*$$

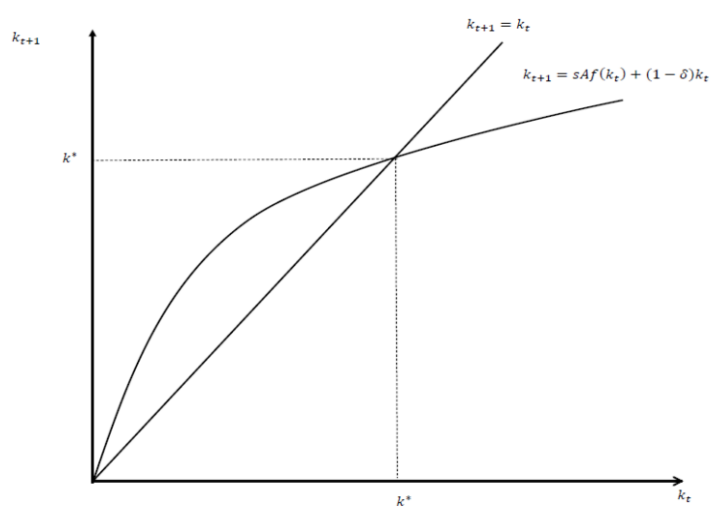

Figure 1. Graph of k_{t+1} against k_t

- Golden rule level of saving
  - Steady state (long-run) consumption is

$$c^* = (1-s)Af(k^*)$$

  - The FOC with respect to saving rate

$$\frac{dc^*}{ds} = -Af(k^*) + (1-s)Af'(k^*)\frac{dk^*}{ds} = 0$$

  - The capital per worker satisfying the above FOC is

$$Af'(k^{gr}) = \delta$$

  - One can derive the Golden rule level of saving (s^{gr}) from the following equation

$$s^{gr} = \frac{\delta k^{gr}}{Af(k^{gr})}$$

1.2 Exercise: Solow Growth Model - Cobb Douglas Case

Suppose the production function in the economy is given as the following Cobb-Douglas form. Assume $N_t = 1$ for all t. Capital stock depreciates at rate δ. A representative household saves constant fraction of output every period with saving rate s. TFP is given by a constant level A.

$$Y_t = AK_t^{\alpha}N_t^{1-\alpha}, \quad \alpha \in (0,1)$$

1. Derive output, consumption and investment per worker as a function of capital per worker.

   Answer:
   Here per worker variables are denoted by lowercase letters.

   $y_t = Ak_t^{\alpha}$
   $c_t = (1-s)y_t = (1-s)Ak_t^{\alpha}$
   $i_t = sy_t = sAk_t^{\alpha}$

2. Calculate the steady state level of capital, output and consumption per worker.

   Answer:
   Steady-state level of capital (k^*) solves the following equation.

$$k^* = sAk^{*\alpha} + (1-\delta)k^*$$

Steady-state capital is $k^* = (\frac{sA}{\delta})^{\frac{1}{1-\alpha}}$*. Using this one can derive steady-state output and consumption level as follows;*

$y^* = Ak^{*\alpha} = A(\frac{sA}{\delta})^{\frac{\alpha}{1-\alpha}}$
$c^* = (1-s)Ak^{*\alpha} = (1-s)A(\frac{sA}{\delta})^{\frac{\alpha}{1-\alpha}}$

3. Assume that TFP increases to A' due to the technology innovation ($A' > A$). An economist predicts the golden rule of saving in this economy would decrease. Do you agree or disagree with her argument?

   Answer:
   k^{gr} *solves the following equation;* $\alpha A k^{\alpha-1} = \delta$*. Then one can get*

   $$k^{gr} = (\frac{\alpha A}{\delta})^{\frac{1}{1-\alpha}}$$

   Plugging this into the steady-state capital level (from Question 2.) and solving for s *give the golden rule level of saving* s^{gr}*;*

   $$s^{gr} = \alpha$$

   It is equal to the capital share of output. In other words, it is invariant to TFP. Therefore her statement is not true under Cobb-Douglas production function.