Econ 702 - Week 6

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1 Exercise

1.1 Uncertainty with Quadratic Utility

Assume that there is uncertainty about next period income, i.e. $Y_{t+1} = Y_{t+1}^H$ with probability 0.5 and $Y_{t+1} = Y_{t+1}^L$ with probability 0.5 and $Y_{t+1}^L < Y_{t+1}^H$. The household maximizes the following lifetime expected-utility

$$U = u(C_t) + \beta\left(\frac{1}{2}U(C_{t+1}^H) + \frac{1}{2}U(C_{t+1}^L)\right) \quad \text{where} \quad u(C_t) = C_t - \frac{\theta}{2}C_t^2$$

such that

$$C_t + S_t = Y_t$$
$$C_{t+1}^H = Y_{t+1}^H + (1 + r_t)S_t, \quad C_{t+1}^L = Y_{t+1}^L + (1 + r_t)S_t.$$

1. Derive the Euler equation.

2. Assume $\beta = 1$ and $r_t = 0$. Solve for C_t as a function of $Y_t, Y_{t+1}^H, Y_{t+1}^L$.

3. Does the household engage in precautionary saving? If not, why?

1.2 Log utility with Borrowing Constraint

Let's assume the household maximizes a lifetime utility function

$$U = ln(C_t) + \beta ln(C_{t+1})$$

such that

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}.$$

1. Derive the Euler equation.

2. Assume $\beta = 1$ and $r_t = 0$. Solve C_t as a function of Y_t, Y_{t+1}. What's the effect of increase in Y_t on the current consumption C_t?

Keep assuming $\beta = 1$ and $r_t = 0$. And suppose that the household faces borrowing constraint

$$C_t \leq Y_t,$$

and the first period income is less than the second period income

$$Y_t < Y_{t+1}.$$

3. Show that the borrowing constraint binds. What is the optimal consumption C_t in this case?

4. Suppose there is an increase in Y_t (but still Y_t is less than Y_{t+1}). What's the effect of increase in Y_t on the current consumption C_t?