The Impact of Uncertainty Shocks on the Investment of Small and Large Firms: Micro Evidence and Macro Implications

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Abstract

This paper explores the heterogeneous impact of uncertainty on firms' investment and its aggregate implications. Empirically, we find that large firms show less investment decline in times of heightened uncertainty. We provide empirical evidence for the underlying driver of the observed size effect: the heterogeneous responses across firms are in fact the consequence of large firms operating in multiple markets rather than their size *per se*. To interpret these findings, we build a heterogeneous firm model with single- and multi-unit firms subject to (i) unit-level real frictions, i.e., fixed and convex investment adjustment costs and (ii) firm-level financial frictions, i.e., costly equity issuance. In our model, even in the absence of diversification, large firms are less responsive to increases in uncertainty because they utilize internal capital markets. Furthermore, we find that the aggregate impact of uncertainty largely depends on heterogeneous responses.

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1 Introduction

How uncertainty affects economic activities has been a long-lasting question and has recently drawn particular attention under the COVID-19 pandemic. The general consensus in the literature is that uncertainty has a negative impact on economic activities, especially investment. Rich evidence from existing studies shows the impact of uncertainty shocks on aggregate investment (Bloom et al. (2018)) and the average impact on firm-level investment (Leahy and Whited (1995), Bloom, Bond and Van Reenen (2007)). However, the literature has not paid much attention to how the impact of uncertainty shocks is systematically different across firms and the underlying reasons for this difference. In this paper, we fill the gap in the literature by investigating how uncertainty influences the investment of small and large firms and discuss the aggregate implications. Firm size is one of the most essential firm-level characteristics because both small and large firms always coexist within an economy. At the same time, the distribution of small and large firms shows substantial differences across countries and times. Therefore, examining the differential impact of uncertainty shock is important because (i) it helps to understand the transmission mechanism of uncertainty to investment choice, and (ii) we can precisely evaluate the overall impact of uncertainty shocks on aggregate investment in a given country and at a given time.

The key questions in this paper are as follows: (i) Do small and large firms respond differently to an increase in uncertainty? (ii) If so, what is the potential source of the discrepancy? (iii) What is the macroeconomic consequence due to the heterogeneity in investment decisions? We address these questions in two steps. First, using micro firm-level data, we document empirical findings that small firms reduce their investment more than large firms in times of heightened uncertainty. We provide empirical evidence for the underlying mechanism of the observed size effect – the number of markets a firm operating in. Second, we build a heterogeneous firm model with single and multi-production unit firms to account for the findings and discuss the aggregate implications.

We start by conducting an empirical analysis using detailed U.S. firm-level Compustat data. This dataset allows us to investigate relatively high-frequency long-panel data, which helps to precisely estimate the size effect. Furthermore, rich balance-sheet information enables us to uncover the size effect. The uncertainty measure is based on the cross-sectional dispersion of the unexpected component of the industry-level output growth rate over the economy following Bloom et al. (2018). In the baseline analysis, we estimate how the semi-elasticity of investment with respect to uncertainty varies with firm size. We find that a firm whose size is one standard deviation larger than that of the average firm is one-third less responsive to increase in uncertainty. This result is robust under various specifications. In particular, we control the interaction between size and another set of aggregate variables to alleviate the concern that the observed size effect merely reflects the excess cyclicality of small firms documented by Crouzet and Mehrotra (2020) due to the highly countercyclical nature of uncertainty.

Based on the previous finding, we proceed to uncover the observed size effect. We find that the heterogeneous responses across firms are in fact the consequence of large firms operating in multiple markets rather than their size *per se*. That is, once we control the number of lines of business: a variable capturing the number of markets a firm access to, firm size loses its significance in explaining heterogeneous responses. However, the semi-elasticity of investment to uncertainty significantly depends on the number of lines of business such that a firm with one more line of business than the average firm is less responsive by 35%. We control another set of variables that might explain the size effect. In particular, a higher firm borrowing cost due to the increase in default risk is one of the important channels through which uncertainty has a real effect. Since large firms or firms operating in multiple markets tend to have a lower level of default probability, the size effect might reflect the default risk channel. In this regard, we control firm-level leverage and "distance to default", which are known to capture firm-level default risk (Ottonello and Winberry (2020)). However, we do not find any evidence that the observed size effect is associated with firm-level default probability or debt burden.

To interpret the empirical findings and discuss the aggregate implications, we build a standard general equilibrium heterogeneous firm model with two extensions. First, firms are allowed to choose the number of production units when they enter the market. The unit can be interpreted as a different line of business, a different factory, or a different geographical market as long as the unit needs its own production inputs and faces a certain degree of idiosyncratic shocks. Second, each firm faces two types of frictions. At the unit level, a firm has to incur fixed and convex adjustment costs upon non-zero investment. At the firm level, if a firm decides to raise funds from the external financial market, it has to pay a finance cost as in Gomes (2001). Due to firm-level financial frictions, the boundary of the firm has an important implication for firm-level investment behavior.

In the model, multi-unit firms are less responsive to uncertainty shocks, and most of the dampened effect is associated with the interaction between the real options channel and the inter-dependence of investment within a multi-unit firm. The inter-dependence arises from the real and financial frictions that cause a multi-unit firm to give up simultaneously investing in both units. Under this situation, even though a multi-unit firm has a good investment opportunity in one unit, it is sometimes willing not to invest because the other unit has a better opportunity and internal funds are limited. Then, how does this interrelationship alleviate the impact of uncertainty shocks? An increase in uncertainty causes firms to initially pause their investment in both units through the real options channel. At the same time, the initial reduction of investment in one particular unit enlarges internal funds and has a positive effect on the other unit's investments, and vice versa. Hence, the initial decrease in investment through the real options channel is partially offset by the positive effect of relaxing financial constraints. Obviously, multi-unit firm diversification also has a dampening effect, but we find that most differences are explained by the inter-dependence effect.

We calibrate the model to match the standard moments in the literature. The model generates the nontargeted moments from the empirical analysis reasonably well. Then, we study the aggregate implication of firm-level heterogeneity by contrasting the benchmark economy with the counterpart economy, which has only single-unit firms. The goal of this analysis is to investigate the contribution of a multi-unit firm's dampened response to aggregate investment fluctuation due to uncertainty shocks. We find that the presence of a multi-unit firm has an adverse effect on a single-unit firm's response because the general equilibrium-smoothing effect is less favorable to single-unit firms in the benchmark economy. However, overall, a multi-unit firm helps to mitigate the negative effect of uncertainty shocks on aggregate investment. This result arises from the fact that multi-unit firms account for a significant portion of the economy and the dampened effect of the multi-unit firm's response is large. This result suggests that the role of heterogeneity crucially depends on the adjustment of the market price, especially the real interest rate, and the distribution of firms.

The remainder of the paper is organized as follows: Section 1 provides information on the related literature. Section 2 provides micro empirical evidence. Section 3 describes the structural model to address the main question. Section 4 presents how we choose parameter values in the model. Section 5 studies the main underlying mechanism in the model and Section 6 explores the aggregate implications. Section 7 concludes.

2 Literature review

This paper contributes to three broad streams of literature. The first stream explores the role of uncertainty shocks over the business cycle. Several related works uncover the effect of uncertainty shocks using a structural general equilibrium model. Bachmann and Bayer (2013) and Bloom et al. (2018) show the macroeconomic implications of uncertainty shocks using a framework under real frictions, i.e., non-convex adjustment costs, in a frictionless financial market environment. Another set of papers focuses on the financial friction channel through which uncertainty shocks affect real variables. Arellano, Bai and Kehoe (2019) explore the effect of volatility shocks on the labor market under an incomplete financial market with default risk. In a similar spirit, Gilchrist, Sim and Zakrajšek (2014) emphasize the role of financial frictions due to default risk. Christiano, Motto and Rostagno (2014) provide a linearized model to explore the impact of volatility shocks and show that a significant portion of business cycle fluctuations in the U.S. can be explained by volatility shocks. Unlike the papers listed above, Alfaro, Bloom and Lin (2018) argue that both real and financial frictions are important. They show that the interaction between these frictions indeed amplifies the effect of uncertainty shocks under partial equilibrium. Our model is closely related to Alfaro, Bloom and Lin (2018) in terms of the frictions imposed, but we investigate the general equilibrium implications and distinguish single and multi-unit firms. We contribute to the literature by showing that the effect of uncertainty shocks significantly relies on certain characteristics of firms, i.e., the number of production units, and the aggregate implications due to the firm's heterogeneous responses.

Second, this paper contributes to the literature on small and large firms' business cycle fluctuations. Ghosal and Ye (2015) and Ghosal and Loungani (2000) show that an industry that is more populated by small firms tends to respond more to uncertainty shocks in terms of investment and employment. Gertler and Gilchrist (1994) show that small firms respond more to monetary policy shocks than larger firms by focusing on the Romer-Romer episodes. Extending the dataset used by Gertler and Gilchrist (1994), Chari, Christiano and Kehoe (2007) argue that the average cyclical behavior of small firms is roughly the same as that of large firms in more general recession episodes other than the Romer-Romer dates. Based on the same dataset, Kudlyak and Sanchez (2017) show that large firms' short-term debt and sales contracted relatively more than those of small firms during the 2008 financial crisis. Recently, Crouzet and Mehrotra (2020) show that the top 1% of large firms are less cyclically sensitive than the bottom 99% of smaller firms and that the industry scope of the largest firms is associated with the size effect. We extend this literature by showing the differential impact of uncertainty shocks on small and large firms and by providing an underlying mechanism–empirically and theoretically–to explain the size effect.

Third, this paper contributes to the literature that studies the implications of firms' boundaries on investment decisions. Matvos and Seru (2014) show that resource allocation within diversified firms significantly alleviates the effect of external financial market disruption on investment choice. Giroud and Mueller (2015) show that shocks to one plant propagate to other plants within the same firm by reallocating capital and labor. They show that this interaction is significant only if the firm is financially constrained. Almeida, Kim and Kim (2015) study capital allocation within Korean business groups (*chaebol*) in the aftermath of the 1997 Asian crisis. They show that *chaebol* reallocated the resources from firms with low-growth opportunity to those with high-growth opportunity, which helps to mitigate the effect of financial disruption. Kehrig and Vincent (2019) show that among multi-plant firms, most of the variation in the plant-level investment rate occurs within a firm rather than between firms. They argue that in the presence of real and financial frictions, dispersion within a firm results from optimizing behavior and will improve firm performance. We contribute to the literature by showing that the firm's boundaries also play an important role in determining the effect of uncertainty shocks on investment.

3 Empirical analysis

In this section, we address the following questions: (i) Are small firms more responsive to uncertainty than large firms? (ii) If so, what characteristics of small and large firms make the discrepancy? Detailed U.S. firm-level Compustat data are used to address the above questions. This dataset allows us to investigate high-frequency long-panel data, which help to precisely estimate the size effect. Furthermore, rich balance sheet data, which are merged with two other datasets, i.e., Compustat Segment and CRSP, enable us to distinguish the size effect from effects due to another dimension of heterogeneity.

3.1 Data description

3.1.1 Firm-level variables

The main dependent variable is $\Delta log k_{i,t+1}$, where $k_{i,t+1}$ is the book value of the tangible capital stock of firm i at the end of period t, which is deflated by the nonresidential fixed investment good deflator. We use the change in capital stock rather than investment rates based on capital expenditure because micro-level investment is known to be lumpy and erratic, which poses a challenge to precisely estimating the systematic differences in investment behavior across firms and times, as noted by Jeenas (2018). The log of real sales and real book value of total assets are used as proxies for firm size. Another set of variables capturing the firm-level characteristic consists of liquidity, the real sales growth rate, current-assets-to-total-assets ratio, the sales-to-capital ratio, leverage, the distance to default, and the number of markets a firm operating in (the number of lines of business). The main firm-level characteristics other than firm size are leverage, the distance to default and the number of lines of business because they are potential candidates that might explain the observed size effects. The detailed reasons for the choice of these variables are described in the following section. Total debt to total asset ratio is used for the leverage measure. To calculate the distance to default, we merge Compustat with CRSP data and follow Bharath and Shumway (2008) for data processing. The information on the number of lines of business is drawn from Compustat Segment data and calculated following Decker, D'Erasmo and Moscoso Boedo (2016).[1] The sample period is from 1987Q1 to 2017Q4, and all firms in Compustat are used for the analysis except those in finance, insurance, real estate and public administration sectors. The data are cleaned and constructed based on the standard practice in the investment literature, following Ottonello and Winberry (2020). Details on the data cleaning and construction process are available in the appendix. Table 1 presents the summary statistics of the main variables used in the empirical analysis.

Panel (i) in Table 1 shows the marginal distribution of selected firm-level variables, and Panel (ii) shows the unconditional pairwise correlations. As we can see in Panel (ii), large firms tend to have more lines of business and a higher value of the distance to default, which justifies their use as potential sources of size effects.[2] Furthermore, the distance to default is negatively correlated with a firm's leverage, which indicates that a higher debt burden implies a higher default risk.

3.1.2 Uncertainty

The uncertainty measure is based on the cross-sectional dispersion of the unexpected component of industry-level output growth rate throughout the economy following Bloom

[1]Since the Compustat Segment data contain only annual frequency information, the annual information on the lines of business is used to fill in the quarterly data within the same calendar year.

[2]Large number of distance to default implies low default risk.

Table 1: Summary statistics

(i) Marginal Distributions

	size (sales)	size (total assets)	$\Delta ln\ k$	lev	lob	dd
Mean	4.02	5.06	0.007	0.28	2.49	3.84
Median	4.17	5.48	0.0005	0.23	2	3.53
Std	2.5	2.45	0.13	0.38	1.58	4.0
Bottom 5%	-0.18	1.59	-0.08	0	1	-0.69
Top 5 %	7.9	9.56	0.12	0.73	6	10.06

(ii) Correlation matrix

	size (sales)	size (total assets)	lob	lev	dd
size (sales)	1.0				
size (total assets)	0.9324	1.0			
lob	0.3220	0.3178	1.0		
lev	-0.017	0.0483	0.0177	1.0	
dd	0.2168	0.2725	0.0758	-0.2200	1.0

Note: Size is the log of real sales or real book value of total assets, $\Delta ln\ k$ is the log change in the real capital stock, lev is the ratio of total debt to total assets, dd is the distance to default measure, and lob is the number of lines of business.

et al. (2018). The main advantage of using this uncertainty measure over using firm-level volatility as a proxy for uncertainty, as in previous works, is that the former uncertainty measure can alleviate potential endogeneity issues because all firms in the economy face the same degree of uncertainty, which is orthogonal to the idiosyncratic firm-level endogenous components driving firm-level volatility shocks.[3] Furthermore, this choice of uncertainty is consistent with the uncertainty in the model. In the structural model, the firm is not identical to the production unit and can own different production units with different productivities. Additionally, the fluctuation of uncertainty is modeled as the time-varying cross-sectional dispersion of tomorrow's unit-level productivities. Since the lines of business that are identified by the industry (SIC 2-digit) in the empirical analysis correspond to the production units in the model, the uncertainty based on the cross-sectional dispersion of industry-level output growth is the most suitable choice.

We identify the uncertainty measure by estimating the following regression for each industry s

$$g^s_{t+1} = \alpha_s + \beta_s g^s_t + \gamma_s Z_t + u_{s,t+1}$$

[3]There might be a concern that each industry faces different level of uncertainty. We deal with this concern by controlling sector-by-time fixed effect.

where g_t^s is the industry s's output growth at time t; Z_t is the observable macro conditions, i.e., GDP growth, the effective federal funds rate, the unemployment rate, and the CPI inflation rate; and $u_{s,t+1}$ is the unforeseen components of the industry output growth rate. $u_{s,t+1}$ consists of common factor f_{t+1} and idiosyncratic factor $\epsilon_{s,t+1}$:

$$u_{s,t+1} = f_{t+1} + \epsilon_{s,t+1}.$$

Since the main focus is the cross-sectional dispersion of industry-specific unforeseen shocks $\epsilon_{s,t+1}$, to back them out, we run a simple panel regression with only time-fixed effects and sector-fixed effects to control not only the common factor but also permanent differences across sectors as follows

$$u_{s,t+1} = \alpha_t + \alpha_s + e_{s,t+1}.$$

Then, we use $e_{s,t+1}$ as an estimate for $\epsilon_{s,t+1}$, calculate the interquartile range of $e_{s,t+1}$ across sectors and use the result as an estimate of uncertainty at time t

$$\sigma_t = IQR_t(e_{s,t+1}).$$

3.2 Heterogeneous response to uncertainty

3.2.1 Baseline results

The baseline regression is

$$\Delta logk_{i,t+1} = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t} \tag{1}$$

where α_i is a firm-level fixed effect, $\alpha_{s,t}$ is a sector-by-time fixed effect, and $Z_{i,t-1}$ consists of the lagged value of firm-level control variables, i.e., leverage, the distance to default, liquidity, the number of lines of business, real sales growth, the current-assets-to-total-assets ratio, size, and the fiscal quarter. The log of real sales and real total assets are used as proxies for size. The lagged value of size and firm-level controls are used to alleviate potential endogeneity issues. The main coefficient of interest is β, which measures how the semi-elasticity of investment $\Delta logk_{i,t+1}$ with respect to uncertainty depends on firm size. In the regression, both size measures are standardized over the entire sample. Hence, the increase in one unit of the size measure can be interpreted as one standard deviation of the size relative to the sample mean. Standard errors are clustered in two ways to account for correlation within firms and within time.

Table 2: Results of the baseline regression

	1	2	3	4
size × unc	0.29**	0.28**	0.25**	0.29**
	(0.13)	(0.14)	(0.11)	(0.13)
unc			-0.77***	-0.70***
			(0.21)	(0.20)
Time × sector fixed effect	yes	yes	no	no
Obs	240,724	240,724	240,724	240,724
R^2	0.1127	0.1126	0.099	0.099
Size measure	sales	total asset	sales	total asset

Note: Column 1 and 2 show the results from estimating $\Delta logk_{i,t+1} = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t}$ and column 3 and 4 show the results from estimating $\Delta logk_{i,t+1} = \alpha_i + \alpha_{s,q} + \gamma Unc_t + \beta size_{i,t-1} \times Unc_t + \Gamma_1' Z_{i,t-1} + \Gamma_2' Y_t + \epsilon_{i,t}$, where α_i is a firm fixed effect, $\alpha_{s,t}$ is a time by sector fixed effect, $Z_{i,t-1}$ consists of the lagged value of firm-level control variables, i.e., leverage, the distance to default, liquidity, the number of lines of business, real sales growth, the current-assets-to-total-assets ratio, size, and the fiscal quarter. Standard errors in parentheses are two-way clustered by firm and time. For the latter regression, we also include Y_t consisting of real GDP growth rate, monetary policy rate, CPI-based inflation rate, and unemployment rate. We standardize the size measure over the entire sample. The sample period is from 1987Q1 to 2017Q4, and all firms in Compustat are used for the analysis except those in finance, insurance, real estate and public administration sectors. $*$, $**$, and $***$ indicate that the coefficient estimate is significantly different from zero at 10%, 5%, and 1% significance level, respectively, based on standard normal critical values for the two-sided test.

Columns 1 and 2 in Table 2 show the results of the baseline regression. The coefficient estimate of the cross-product term is positive and statistically significant in both size measure specifications. Controlling the time by sector fixed effects, we can interpret that large firms reduce their investment less than smaller firms in times of heightened uncertainty. Since the results from different size measures give very similar results, we focus on sales in the following analysis.[4]

To investigate the average effect of uncertainty on a firm's investment, the sector-by-time fixed effect is omitted, and the following regression is estimated with uncertainty series and

[4]The observed size effect might just reflect the excess cyclicality of small firms documented by Crouzet and Mehrotra (2020) due to the highly countercyclical nature of uncertainty. In the appendix, we perform extra exercises to control for the interaction of lagged size with the other aggregate variables to deal with the concern of the strong countercyclical nature of uncertainty.

other sets of aggregate variables.

$$\Delta logk_{i,t+1} = \alpha_i + \alpha_{s,q} + \gamma Unc_t + \beta size_{i,t-1} \times Unc_t + \Gamma_1' Z_{i,t-1} + \Gamma_2' Y_t + \epsilon_{i,t} \quad (2)$$

where $\alpha_{s,q}$ is the sector-by-quarter fixed effects to control for seasonality and aggregate control variables Y_t consisting of the real GDP growth rate, monetary policy rate, CPI-based inflation rate, and unemployment rate. Other control variables are the same as in the previous regression (1). Columns 3 and 4 in Table 2 report the regression results. Consistent with the existing literature as in Bloom (2009), the average impact of increase in uncertainty is estimated to be negative and statistically significant. The average investment semi-elasticity is -0.77 in response to a one-percentage-point increase in uncertainty. The cross-product term is still positive and statistically significant such that if the firm size is one standard deviation larger than that of the average firm, the investment semi-elasticity of the larger firm increases by 0.25, which is approximately one-third of the average firm's response.

3.2.2 Deciphering mechanism

In the following analysis, we provide evidence for the underlying mechanism of the observed size effect. Based on the positive correlation between firm size and the number of lines of business, the following empirical analysis shows that the observed size effect is explained mostly by the number of lines of business. To rule out other possibilities, we also control the firm-level leverage ratio and distance to default measure. The choice of these controls is motivated by the theory in the literature–a higher firm borrowing cost due to an increase in default risk is an important channel through which uncertainty shocks affect real variables (Arellano, Bai and Kehoe (2019), Gilchrist, Sim and Zakrajšek (2014) and Christiano, Motto and Rostagno (2014)). Since firm size is also highly correlated with the firm's default risk, as shown in Table 1, the observed size effect might represent the default risk channel. However, as is evident in the following regression analysis, the default channel does not seem to be successful in explaining the observed size effects. We provide further evidence that extensive margin adjustment plays an important role in explaining the differential responses, thus suggesting that the 'wait-and-see' effect is asymmetric across firms.

First, we run the following version of regressions to uncover the size effect:

$$\Delta logk_{i,t+1} = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t + \beta_\nu \nu_{i,t-1} \times Unc_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t}. \quad (3)$$

Table 3: Results of regression (3) - Deciphering mechanism

	1	2	3	4	5	6
size × unc	0.291**	0.278**	0.263**	0.144	0.091	0.112
	(0.130)	(0.131)	(0.129)	(0.110)	(0.109)	(0.109)
lev × unc		0.081			0.131	0.129
		(0.078)			(0.084)	(0.086)
dd × unc			0.155**		0.186**	-0.074
			(0.077)		(0.082)	(0.126)
lob × unc				0.253***	0.252***	0.256***
				(0.06)	(0.061)	(0.064)
unc						-0.71***
						(0.298)
time × sector fixed effect	Yes	Yes	Yes	Yes	Yes	No
obs	240,724	240,724	240,724	240,724	240,724	240,729
R^2	0.1127	0.1127	0.1127	0.1129	0.1130	0.0999

Note: Column 1 repeats the results estimating $\Delta logk_{i,t+1} = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t}$, where α_i is a firm fixed effect, $\alpha_{s,t}$ is a time by sector fixed effect, $Z_{i,t-1}$ consists of the lagged value of firm-level control variables, i.e., leverage, the distance to default, liquidity, the number of lines of business, real sales growth, the current-assets-to-total-assets ratio, size, and the fiscal quarter. In addition, column 2 includes leverage (lev), column 3 includes the distance to default (dd), column 4 includes the number of lines of business (lob) and column 5 includes all. In column 6, the time by sector fixed effect is dropped but quarter by sector fixed effects and several aggregate variables - real GDP growth rate, monetary policy rate, CPI-based inflation rate, and unemployment rate - as well as uncertainty are controlled. Standard errors in parentheses are two-way clustered by firm and time. We standardize the size, leverage and the distance to default over the entire sample. For the number of lines of business, we subtract it by the average of the entire sample but do not divide it by the standard deviation. The sample period is from 1987Q1 to 2017Q4, and all firms in Compustat are used for the analysis except those in finance, insurance, real estate and public administration sectors. $*$, $**$, and $***$ indicate that the coefficient estimate is significantly different from zero at 10%, 5%, and 1% significance level, respectively, based on standard normal critical values for the two-sided test.

The main difference between this version of the regression and the baseline regression (1) is the additional inclusion of the cross-product of uncertainty with a variable of interest, i.e., $\nu_{i,t-1}$, which is leverage (lev), the distance to default (dd) or the number of lines of business (lob). For each variable of interest, a different regression is performed with a different cross-product term. The leverage and the distance to default are chosen to capture the default risk as in Ottonello and Winberry (2020). As in Decker, D'Erasmo and Moscoso Boedo (2016) and Matvos and Seru (2014), the number of lines of business (identified by two-digit SIC codes) is used as the proxy for the number of production units each firm owns.

Table 3 shows the results. Column 1 repeats the results of the baseline regression (1) for ease of comparison. Columns 2 and 3 show the regression results when the leverage and the distance to default are controlled, respectively. However, their inclusion does not seem to change the coefficient estimate of the size effect relative to the baseline results in Column 1. Column 4 shows the results with the number of firm production units. Compared to the default-level proxies, the number of production units significantly affects the coefficient estimate of the size effect. Column 5 includes all variables, and column 6 includes all variables but dropping sector and time fixed effects to examine the average effect of uncertainty, as in regression (2). The overall results suggest that the size effect is not driven mainly by the default-risk mechanism and that the number of production units has important implications for the observed size effect.

3.2.3 Ex-post behavior – ruling out the default risk channel

Focusing on ex-ante firm-level heterogeneity would not be enough to rule out the possibility of the default-risk channel because there might be unobserved firm-level characteristics that covariate with firm size but cause the default risk of small firms to increase more than that of large firms. Alternatively, the number of production units would be a better proxy for a firm's default risk because if a firm owns more production units, it might be perceived as well diversified. In that case, a small firm's (stand-alone firm's) borrowing costs increase more, and hence, the small firm's (stand-alone firm's) investment drops more. To rule out this possibility, we investigate the ex-post change in default risks and financial variables. Similar to equation (2), the following regression is performed:

$$\Delta y_{i,t} = \alpha_i + \alpha_{s,q} + \gamma Unc_t + \beta size_{i,t-1} \times Unc_t + \Gamma_1' Z_{i,t-1} + \Gamma_2' Y_t + \epsilon_{i,t}, \tag{4}$$

where $\Delta y_{i,t}$ equals three variables, namely, the change in the distance to default, the change in short-term debt, and the change in long-term debt.[5]

As we can see in Table 4, the coefficients on uncertainty are estimated to be statistically significant except those for the short-term debt, and all of them are estimated to be negative. Hence, uncertainty has a negative impact on all the dependent variables on average. However, the coefficient estimates on the cross-product between size and uncertainty turn out to be statistically insignificant in all cases. Furthermore, the signs of estimates are at odds

[5]A change in the distance to default is normalized by its own lagged value, and changes in real short-term and long-term debt are normalized by the lagged value of total assets.

Table 4: Results of regression (4) - Ex-post behavior

dep var		Δ dd			Δ short-term debt			Δ total debt	
	1	2	3	4	5	6	7	8	9
unc	-5.967*	-5.845*	-5.840*	-0.039	-0.036	-0.023	-0.172*	-0.166*	-0.194*
	(3.269)	(3.282)	(3.303)	(0.032)	(0.032)	(0.031)	(0.090)	(0.092)	(0.105)
size × unc		-0.654	-0.748		-0.019	-0.017		-0.035	-0.042
		(0.752)	(0.766)		(0.022)	(0.024)		(0.027)	(0.026)
lob × unc			0.091			-0.006			0.023
			(0.414)			(0.006)			(0.015)
obs	237,339	237,339	237,339	239,791	239,791	239,791	239,715	239,715	239,715
R^2	0.0357	0.0357	0.357	0.0263	0.0263	0.0263	0.0663	0.0663	0.0663

Note: The table shows the results from estimating $\Delta y_{i,t} = \alpha_i + \alpha_{s,q} + \gamma Unc_t + \beta size_{i,t-1} \times Unc_t + \Gamma_1' Z_{i,t-1} + \Gamma_2' Y_t + \epsilon_{i,t}$, where α_i is a firm fixed effect, $\alpha_{s,t}$ is a time by sector fixed effect, $Z_{i,t-1}$ consists of the lagged value of firm-level control variables, i.e., leverage, the distance to default, liquidity, the number of lines of business, real sales growth, the current-assets-to-total-assets ratio, size, and the fiscal quarter, and Y_t consists of real GDP growth rate, monetary policy rate, CPI-based inflation rate, and unemployment rate. The dependent variable $\Delta y_{i,t}$ is (i) change of the distance to default in column 1 to 3, (ii) change of real short-term debt in column 4 to 6 and (iii) change of real total-debt in column 7 to 9. Standard errors in parentheses are two-way clustered by firm and time. We standardize the size over the entire sample. For the number of lines of business, we subtract it by the average of the entire sample but do not divide it by the standard deviation. The sample period is from 1987Q1 to 2017Q4, and all firms in Compustat are used for the analysis except those in finance, insurance, real estate and public administration sectors. $*$, $**$, and $***$ indicate that the coefficient estimate is significantly different from zero at 10%, 5%, and 1% significance level, respectively, based on standard normal critical values for the two-sided test.

with the idea that the size effect represents the default risk channel because negative coefficients on the cross-product terms imply that a large firm's default risk or financial variables respond more to uncertainty. We also include the interaction of the number of lines of business and uncertainty in Columns (3), (6), and (9), but the coefficients are not significantly different from zero. Overall, the findings in Table 4 seem inconsistent with the view that the observed size effect is driven mainly by the default risk channel.

3.2.4 Ex-post behavior – supporting the real options channel

In the following analysis, we show additional evidence consistent with the idea that the observed size effect is closely related to the asymmetric 'wait-and-see' effect across different firms. Rather than relying on the default risk channel, a set of papers in the literature focuses on the real options mechanism through which uncertainty has a negative impact on a firm's investment via 'wait-and-see' effects (Bloom (2009), Bloom et al. (2018) and Bachmann and

Bayer (2013)). That is, firms are more cautious about their investment in times of heightened uncertainty, so they postpone new investment projects until the uncertainty is resolved. This implies the extensive margin adjustment plays an important role in determining the firm's investment decision. Hence, if the real options channel drives the observed size effect, small and large firms must show different patterns in terms of an extensive margin as well as an intensive margin choice. The following version of the regression is performed to confirm the prediction:

$$\begin{aligned} \mathbf{I}(i_{j,t}/k_{j,t-1} > 0.05) = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t \\ + \beta_{LoB} LoB_{i,t-1} \times Unc_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t} \end{aligned} \tag{5}$$

where $i_{j,t}$ is the real capital expenditures, $k_{j,t-1}$ is the lagged value of real tangible capital, and the dependent variable $\mathbf{I}(i_{j,t}/k_{j,t-1} > 0.05)$ is the indicator variable, which takes a value of 1 if the firm's investment rate $i_{j,t}/k_{j,t-1}$ is larger than 5 percent or 0 otherwise. Here, we focus on large investment change, which is called *investment spikes* in the literature rather than inaction with zero investment because identifying inaction precisely at the micro level is a difficult task due to the substantial heterogeneity (i) in capital assets with associated heterogeneity in the depreciation rate and adjustment costs and (ii) in the types of investment episodes (e.g., maintenance vs. large new projects), as noted by Cooper and Haltiwanger (2006).[6]

Table 5 shows the results. Column 1 reports the results with the size effect. Consistent with the prediction of the real options channel, if firm size is one standard deviation larger than the average firm, the probability of investment spikes increases by 1.2 percent in times of heightened uncertainty. Column 2 reports the result with lines of business. If a firm owns one more line of business relative to the average number, the probability of investment spikes increases by 1.4 percent in response to increase in uncertainty. Column 3 shows the regression results when both the size and the number of lines of business are controlled. The inclusion of the number of lines of business significantly alters the coefficient estimate of the size effect, but that of lines of business is barely affected. Column 4 drops sector by time fixed effects to examine the average effect of uncertainty. The average effect of uncertainty is estimated to be negative and statistically significant, and the effect is weaker for multi-

[6]The choice of a 5 percent threshold is standard in the literature (20 percent in the annual horizon). However, the result is not sensitive to thresholds from 3 to 15 percent.

Table 5: Results of regression (5) - Extensive margin adjustment

	1	2	3	4
size $\times$ uncertainty	1.204***		0.431	0.428
	(0.398)		(0.473)	(0.504)
lob $\times$ uncertainty		1.422***	1.335***	1.26***
		(0.323)	(0.370)	(0.371)
uncertainty				-1.63**
				(0.86)
time $\times$ sector fixed effect	yes	yes	yes	no
Observations	235,695	235,695	235,695	235,700
R^2	0.3404	0.3406	0.3406	0.3283

Note: Results from estimating $\mathbf{I}(i_{j,t}/k_{j,t-1} > 0.05) = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t + \beta_{LoB} LoB_{i,t-1} \times Unc_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t}$ where α_i is a firm fixed effect, $\alpha_{s,t}$ is a time by sector fixed effect, $Z_{i,t-1}$ consists of the lagged value of firm-level control variables, i.e., leverage, the distance to default, liquidity, the number of lines of business, real sales growth, the current-assets-to-total-assets ratio, size, and the fiscal quarter. Column 1 includes size, column 2 includes the number of lines of business (lob) and column 3 includes all. In column 4, the time by sector fixed effect is dropped but quarter by sector fixed effects and several aggregate variables - real GDP growth rate, monetary policy rate, CPI-based inflation rate, and unemployment rate - as well as uncertainty are controlled. Standard errors in parentheses are two-way clustered by firm and time. We standardize the size over the entire sample. For the number of lines of business, we subtract it by the average of the entire sample but do not divide it by the standard deviation. The sample period is from 1987Q1 to 2017Q4, and all firms in Compustat are used for the analysis except those in finance, insurance, real estate and public administration sectors. $*$, $**$, and $***$ indicate that the coefficient estimate is significantly different from zero at 10%, 5%, and 1% significance level, respectively, based on standard normal critical values for the two-sided test.

unit firms. Therefore, the overall results suggest that the size effect reflects the real options channel rather than the default risk mechanism and that the real options effect is weaker for larger firms because they operate in multiple production units. In the following section, we build up a structural model to explain the asymmetric real options effects on small and large firms based on the empirical findings.

4 Model

The model builds on Bachmann and Bayer (2013) and Bloom et al. (2018), who study the impact of uncertainty shocks based on the 'wait-and-see' mechanism under the general equilibrium framework. The economy consists of three types of agents: a representative household, single-unit firms, and multi-unit firms. The household consumes final goods, owns firms and supplies labor. Firms operate either single- or multi-production units. The units can be interpreted as different factories, different geographic markets, or different business segments or product lines within a firm as long as they need their own input for the production process and face a certain degree of idiosyncratic shocks, such as demand or productivity shocks. Firms are able to choose the number of units when they enter the market.[7] They hire labor to produce final goods and accumulate capital, which is subject to fixed and convex investment adjustment costs. The main departure of this model from the standard wait-and-see literature is a wedge between internal and external funds. In particular, when firms do not have enough funds to finance their investment project from their profit, they are able to issue new equity by paying finance costs, as in Gomes (2001). To focus on the friction between internal and external funds, we abstract from the distinction between debt and equity financing.

4.0.1 Physical environment

The economy consists of a unit measure of firms that can choose the number of units to operate upon the entry. A multi-unit firm is assumed to own 2 different production units for numerical tractability. A large number of production units exponentially increases the computation burden due to the curse of dimensionality without adding any economic intuition.

For any given period, $\pi_N \in (0, 1)$ new firms enter the economy, and each firm draws a fixed cost γ from the distribution $F(\gamma)$. The random fixed cost γ follows i.i.d across firms and time. Given this cost, they decide whether to be multi-unit firms by paying it in the labor unit. Otherwise, the firm will be a single-unit firm without any cost. The initial state and optimization problem of entrants will be described below. To keep the measure of firms constant, it is assumed that each firm faces a fixed exit probability, π_N, that it will be forced to exit the market after production in each period.

Each production unit is featured by Cobb-Douglas decreasing-returns-to-scale technology

$$y = Azk^{\alpha}n^{\nu}, \quad 0 < \alpha + \nu < 1,$$

[7]Firms are not allowed to change the number of units once they choose. This assumption is based on the idea that firm's decision making on organizational structure is less relevant in the business cycle frequency (quarterly frequency in this paper). Futhermore, we investigate uncertainty shocks which show pretty transitory nature. In the appendix, we show that indeed, there is no systemetic relationship between uncertainty and firm's choice on the number of units in the data.

where A is an aggregate TFP following the AR(1) process, which is common across all units in the economy:

$$lnA' = \rho_A lnA + \epsilon'_A, \quad \epsilon'_A \sim N(0, \sigma_A^2),$$

and z is a unit-level idiosyncratic productivity that also follows the AR(1) process for single-unit firms:

$$lnz' = \rho_z lnz + \epsilon'_z, \quad \epsilon'_z \sim N(0, \sigma_z^2),$$

and follows the VAR(1) process for multi-unit firms:

$$\begin{bmatrix} lnz'_1 \\ lnz'_2 \end{bmatrix} = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_z \end{bmatrix} \begin{bmatrix} lnz_1 \\ lnz_2 \end{bmatrix} + \begin{bmatrix} \epsilon'_1 \\ \epsilon'_2 \end{bmatrix}, \quad \begin{bmatrix} \epsilon'_1 \\ \epsilon'_2 \end{bmatrix} \sim N(0, \Sigma) \quad \Sigma = \begin{bmatrix} \sigma_z^2 & \sigma_{12} \\ \sigma_{12} & \sigma_z^2 \end{bmatrix}.$$

allowing nonzero correlation between different units within a multi-unit firm, which will reflect the firm-level shocks affecting both units within a firm. The standard deviation of future shocks σ_A and σ_z are known at the current period and are time-varying based on a two-state Markov chain.[8] The timing assumption reflects that firms become informed about the distribution of future shocks A' and z' that they will face. Thus, the evolution of σ_A and σ_z broadly captures the uncertainty of tomorrow's business conditions. k is the capital stock, and n is the labor hired for the production of output.

The beginning of the period distribution of single-unit firms over (z, k) is denoted by μ_S and that of multi-unit firms over (z_1, z_2, k_1, k_2) is denoted by μ_L. Therefore, the aggregate TFP A, the volatility of aggregate TFP σ_A, the volatility of unit-specific productivity shocks σ_z and the distributions μ_S and μ_L constitute the aggregate state $\mathbf{S} = \{A, \sigma_A, \sigma_z, \mu_S, \mu_L\}$.

Firms are subject to unit-level real frictions and firm-level financial frictions. Specifically, at the unit level, a firm has to incur fixed and convex adjustment costs upon non-zero investment. Formally,

$$\Phi(k, k') \equiv \frac{\phi_c}{2}\left(\frac{k' - (1-\delta)k}{k}\right)^2 k + \phi_f \mathbb{I}(k' \neq (1-\delta)k). \tag{6}$$

The fixed cost captures the disruptive effect of investment on the production process due to restructuring or installing new capital (Caballero et al. (1995), Cooper and Haltiwanger (2006), Doms, Dunne et al. (1998), and Gourio and Kashyap (2007)). At the firm level, if a firm decides to raise funds from the external financial market via new equity issuance, it

[8] We assume that the correlation across different units within a firm is constant over time so that the covariance is also time-varying.

has to pay a finance cost as in Gomes (2001)

$$\psi(d;\mathbf{S}) = \begin{cases} \psi_1 + \psi_2(\mathbf{S}) \times \mid d \mid \quad if \quad d < 0 \\ 0 \quad if \quad d \geq 0. \end{cases} \tag{7}$$

Due to this extra cost, firms take external sources of funds as a last resource only when the sum of capital adjustment cost and investment exceeds the operating profit. This cost reflects expenditures for both direct costs associated with various information and disclosure requirements and other administrative expenses or indirect costs related to asymmetric information and managerial incentive problems. We also assume that the marginal finance cost $\psi_2(\mathbf{S})$ is a function of aggregate states, especially an increasing function of uncertainty following Alfaro, Bloom and Lin (2018). This assumption is motivated by (i) the predictions from the micro-founded model (Bigio (2015)), which show a negative relationship between uncertainty and financing costs based on adverse selection, and (ii) empirical evidence that uncertainty and external finance costs are highly positively correlated (Caldara et al. (2016)). All adjustment costs are in labor units and rebated to the representative household as a lump sum.

4.0.2 Single-unit firms

At the beginning of the period, aggregate state $\mathbf{S}$ is realized, and a firm starts with predetermined capital stock k and idiosyncratic productivity z. Given the states, firms learn whether they exit after production with probability π_N or keep producing in the next period with probability $1 - \pi_N$. Immediately thereafter, firms hire labor and produce output. If they survive, firms also choose the amount of dividend and investment under the frictions described in the previous section.

Let $V_0^S(z,k;\mathbf{S})$ be an expected value function of a firm just before it realizes whether it will exit or not. Then, it becomes

$$\begin{aligned} V_0^S(z,k;\mathbf{S}) = \pi_N \ \max_n \{Azk^\alpha n^\nu - w(\mathbf{S})n + (1-\delta)k - w(\mathbf{S})\Phi(k,0)\} \\ +(1-\pi_N)V^S(z,k;\mathbf{S}) \end{aligned}$$

where $V^S(z,k;\mathbf{S})$ is the value function of surviving firms. If a firm does not continue, it chooses labor to maximize the current dividend to the representative household. Since it will not carry any capital stock into the future, i.e., $k' = 0$, the dividend of exiting firms consists of operating profits and the undepreciated capital stock minus the adjustment costs.

Surviving firms choose labor to maximize the current profit $Azk^{\alpha}n^{\nu} - w(\mathbf{S})n$. Furthermore, they choose the amount of investment to maximize the present value of dividends to the representative household. Upon nonzero investment, a firm has to pay adjustment costs in the labor unit. In addition, if the firm decides to increase the capital stock but the operating profits are not sufficient to cover the firms' new investment and physical adjustment costs, it raises external finance with finance costs. Due to the wedge between internal and external funds, it is never optimal to issue new equity while paying dividends, i.e., when the operating profits are sufficient to finance the investment project. To simplify the exposition, we define the firm's payout d before financing costs as

$$d = Azk^{\alpha}n^{\nu} - wn - k' + (1-\delta)k - w\Phi(k, k').$$

If d is positive, a firm pays dividends to the household. A negative value of d implies that a firm does not have enough funds to finance its investment project so that it issues new equity. In the absence of financial distortions, the equity issuance $d < 0$ reduces the value of existing shares by the same amount. However, in the model with finance costs, the value of existing shares is reduced by more than the amount of newly issued shares. Given an aggregate state $\mathbf{S}$, an individual state (z, k), and law of motions for the joint distributions μ_S and μ_L,

$$\mu_S' = \Gamma_S(\mathbf{S}), \quad \mu_L' = \Gamma_L(\mathbf{S}),$$

a surviving single-unit firm maximizes the present value of d, net of financing costs $\psi(d; \mathbf{S})$ by solving the following Bellman equation

$$V^S(z, k; \mathbf{S}) = \max_{d,n,k'} \; d - w(\mathbf{S})\psi(d; \mathbf{S}) + E\left[m(\mathbf{S}, \mathbf{S}')V_0^S(z', k'; \mathbf{S}') \mid z; \mathbf{S} \right]$$

where

$$d = Azk^{\alpha}n^{\nu} - w(\mathbf{S})n - k' + (1-\delta)k - w(\mathbf{S})\Phi(k, k'), \tag{8}$$

$\Phi(k, k')$ is defined by (6), finance cost $\psi(d; \mathbf{S})$ is defined by (7), $m(\mathbf{S}, \mathbf{S}')$ is the stochastic discount factor, and $V_0^S(z', k'; \mathbf{S}')$ is the expected value function just before firms realize exit status.[9]

[9]Since all firms are owned by the representative household, the discount factor is $m(\mathbf{S}, \mathbf{S}') = \beta u_c(\mathbf{S}')/u_c(\mathbf{S})$, where β is the time discount factor, $u_c(\mathbf{S}')$ and $u_c(\mathbf{S})$ are the marginal utility of consumption

4.0.3 Multi-unit firms

The timing of a multi-unit firm is the same as that of a single-unit firm. At the beginning of the period, aggregate state $\mathbf{S}$ is realized, and a firm starts with capital stock k_1, k_2 and idiosyncratic productivity z_1, z_2 in each unit. Given the states, firms learn whether they exit or not. Immediately thereafter, they hire labor and produce output. Firms who realize they survive choose the investment to maximize the precent value of dividends to the household.

Let $V_0^L(z_1, z_2, k_1, k_2; \mathbf{S})$ be an expected value function of a multi-unit firm just before it realizes whether it will exit or not. Then, it becomes

$$V_0^L(z_1, z_2, k_1, k_2; \mathbf{S}) = \pi_N \max_{n_1,n_2}\{Az_1k_1^{\alpha}n_1^{\nu} + Az_2k_2^{\alpha}n_2^{\nu} - w(\mathbf{S})(n_1 + n_2)$$
$$+(1-\delta)(k_1 + k_2) - w(\mathbf{S})(\Phi(k_1, 0) + \Phi(k_2, 0))\} + (1 - \pi_N)V^L(z_1, z_2, k_1, k_2; \mathbf{S})$$

where $V^L(z_1, z_2, k_1, k_2; \mathbf{S})$ is the value function of surviving multi-unit firms.

At the unit-level, a multi-unit firm faces the same friction as single-unit firms, i.e., fixed and convex adjustment costs of physical capital. However, due to the firm-level financial frictions, larger boundary of a multi-unit firm plays an important role in determining investment and financing decisions. Specifically, a multi-unit firm can reallocate resources across different units without any frictions. If it does not have enough cash flows generated by unit 1 to finance investment project in the same unit, e.g., $Azk_1^{\alpha}n_1^{\nu} - wn_1 < k_1' - (1-\delta)k_1 + w\Phi(k_1, k_1')$, the firm can reallocate the profit from the other unit to avoid finance costs. On the other hand, if a firm has good investment opportunities in both units, it compares two different scenarios – (i) investing in one unit which gives higher return without relying on the external finance or (ii) investing in both units by raising funds from costly equity issuance – and chooses more profitable one. Hence, the way in which a multi-unit firm is affected by firm-level finance costs is different from that of a single-unit firm, even though they face the same degree of external finance costs. A surviving firm maximizes the present value of d, net of financing costs $\psi(d; \mathbf{S})$. Given an aggregate state $\mathbf{S}$, an individual state (z_1, z_2, k_1, k_2), and law of motions for the joint distributions

$$\mu_S' = \Gamma_S(\mathbf{S}), \quad \mu_L' = \Gamma_L(\mathbf{S}),$$

for current and next period, respectively.

a multi-unit firm solves the following Bellman equation by choosing d, n_1, n_2, k_1', k_2'

$$V^L(z_1, z_2, k_1, k_2; \mathbf{S}) = \max_{d, n_1, n_2, k_1', k_2'} d - w\psi(d; \mathbf{S}) + E\Big[m(\mathbf{S}, \mathbf{S}')V_0^L(z_1', z_2', k_1', k_2'; S') \mid z_1, z_2; \mathbf{S}\Big]$$

where

$$d = Az_1 k_1^\alpha n_1^\nu + Az_2 k_2^\alpha n_2^\nu - w(\mathbf{S})(n_1 + n_2) - (k_1' + k_2') + (1-\delta)(k_1 + k_2) - w(\mathbf{S})\big(\Phi(k_1, k_1') + \Phi(k_2, k_2')\big) \quad (9)$$

and $V_0^L(z_1', z_2', k_1', k_2'; \mathbf{S}')$ is the expected value function immediately before firms realize exit status.

4.0.4 Entrants

Each new firm starts with zero capital stock but is able to issue new equity to finance the investment. Entrants are subject to the capital adjustment cost and finance cost. Upon the entry, new firms draw a fixed cost γ, given which they choose to be single- or multi-unit firms. If a firm chooses to be a single-unit, it will draw the idiosyncratic productivity shock z from an ergodic distribution $G_1(z)$ implied by the AR(1) process of the idiosyncratic productivity shocks to incumbent firms. Given the same aggregate conditions and law of motions for distributions as incumbent firms, the single-unit firm solves

$$V_E^S(z; \mathbf{S}) = \max_{k', d} d - w(\mathbf{S})\psi(d; \mathbf{S}) + E\Big[m(\mathbf{S}, \mathbf{S}')V_0^S(z', k'; \mathbf{S}') \mid z, \mathbf{S}\Big]$$

where $d = -k' - w\phi_f$ and the finance cost $\psi(d; \mathbf{S})$ is defined by (7).[10]

If a firm chooses to be a double-unit firm, it will draw two distinct idiosyncratic productivity shocks z_1 and z_2 from the ergodic distribution $G_2(z_1, z_2)$, which is implied by the VAR(1) process for incumbent firms. Then, the double-unit firm solves

$$V_E^L(z_1, z_2; \mathbf{S}) = \max_{k_1', k_2', d} d - w(\mathbf{S})\psi(d; \mathbf{S}) + E\Big[m(\mathbf{S}, \mathbf{S}')V_0^L(z_1', z_2', k_1', k_2'; \mathbf{S}') \mid z_1, z_2; \mathbf{S}\Big]$$

[10]Since new entrants start with zero capital stock, their convex adjustment cost is assumed to be zero.

where $d = -k_1' - k_2' - 2w\phi_f$ and the finance cost $\psi(d; \mathbf{S})$ is defined by (7) subject to the same aggregate conditions and law of motions for distributions as incumbent firms.

Given the value functions, the measure of firms who choose to be multi-unit will be determined. Define the threshold level of fixed cost $\hat{\gamma}(\mathbf{S})$ as

$$w(\mathbf{S})\hat{\gamma}(\mathbf{S}) \equiv \int V_E^L(z_1, z_2; \mathbf{S}) dG_2(z_1, z_2) - \int V_E^S(z; \mathbf{S}) dG_1(z).$$

New firms with fixed cost $\gamma < \hat{\gamma}(\mathbf{S})$ will choose to be multi-unit firms since

$$\int V_E^L(z_1, z_2; \mathbf{S}) dG_2(z_1, z_2) - \int V_E^S(z; \mathbf{S}) dG_1(z) > w(\mathbf{S})\gamma,$$

i.e., the benefit of being a multi-unit firm is larger than the cost. Therefore, the measure of new firms who choose to operate in multiple unit becomes $Pr(\gamma < \hat{\gamma}(\mathbf{S})) = F(\hat{\gamma}(\mathbf{S}))$.

4.0.5 Household

There is a representative household that chooses the consumption, labor supply, and investment in firm shares to maximize lifetime utility.

$$U(s; \mathbf{S}) = \max_{C, N, s'(z,k), s'(z_1,z_2,k_1,k_2)} ln\ C - \theta N + \beta E\big[U(s'; \mathbf{S}') \mid \mathbf{S}\big], \quad \theta > 0$$

given the following budget constraint

$$\begin{aligned} C &+ (1-\pi_N)\bigg(\int s' p_s\ d\mu_S(z,k) + \int s' p_s\ d\mu_L(z_1,z_2,k_1,k_2)\bigg) \\ &+ \pi_N\bigg(\int s' p_{s,new}\ dG_1(z) + \int s' p_{s,new}\ dG_2(z_1,z_2)\bigg) \\ &= w(\mathbf{S})N + (1-\pi_N)\bigg(\int s(\tilde{d} + p_s)\ d\mu_S(z,k) + \int s(\tilde{d} + p_s)\ d\mu_L(z_1,z_2,k_1,k_2)\bigg) \\ &+ \pi_N\bigg(\int s\ \tilde{d}_{Exit}\ \ d\mu_S(z,k) + \int s\ \tilde{d}_{Exit}\ \ d\mu_L(z_1,z_2,k_1,k_2)\bigg) + Adj_0(\mathbf{S}), \end{aligned}$$

where $w(\mathbf{S})$ is the wage, $Adj_0(\mathbf{S})$ is the income from the adjustment cost by firms, which includes both physical and financial adjustment costs, p_s is the value of stock, s is the share of previous period equity and s' is the share of equity chosen today. For simplicity, the arguments of $\tilde{d}, \tilde{d}_{Exit}, p_s, p_{s,new}, s, s'$ are suppressed (those variables are functions of $(z, k; S)$ or $(z_1, z_2, k_1, k_2; S)$). $\tilde{d}$ is either the dividend payment or effective equity issuance of surviving firms. When $\tilde{d}$ is positive, the household earns dividend payments from the firm so that $\tilde{d} = d$, where d is the firm's payout before finance costs are defined as (8) and (9). However, if $\tilde{d}$ is negative, there is zero dividend to the household. In this case, $\tilde{d} = d - w\psi(d; \mathbf{S}) < 0$ represents the new equity issuance plus the issuance cost, which is the gap between the current value of total equity and that of pre-existing equity. $p_{s,new}$ is the value of new firms, and $\tilde{d}_{Exit}$ is the dividend payments from exiting firms.

4.0.6 Recursive equilibrium

A recursive competitive equilibrium in this economy is defined by a set of quantity functions $\{C, N, s', K'_S, N^d_S, K'_{1,L}, K'_{2,L}, N^d_{1,L}, N^d_{2,L}, K^E_S, K^E_{1,L}, K^E_{2,L}\}$, pricing function $\{w, p_s, p_{s,new}, m\}$, lifetime utility and value functions $\{U, V^S_0, V^L_0, V^S_E, V^L_E\}$, where $\{V^S_0, V^L_0\}$ and $\{K'_S, N^d_S, K'_{1,L}, N^d_{1,L}, , K'_{2,L}, N^d_{2,L}\}$ are the value and policy functions of incumbent single- and multi-unit firms, respectively, and $\{V^S_E, V^L_E\}$ and $\{K^E_S, K^E_{1,L}, K^E_{2,L}\}$ are the value and policy functions of newborn single- and multi-unit firms, while U and $\{C, N^S, s'\}$ are the value and policy functions that solve the household problem. Given the quantity and pricing functions, the (i) goods, (ii) asset, and (iii) labor markets clear with

$$
\begin{aligned}
(i)\ C(S) = &\int Azk^{\alpha} N^d_S(z,k;\mathbf{S})^{\nu}\, d\mu_S(z,k) \\
&+ \int \sum_{i=1,2} Az_i k_i^{\alpha} N^d_{i,L}(z_1,z_2,k_1,k_2;\mathbf{S})^{\nu}\, d\mu_L(z_1,z_2,k_1,k_2) \\
&-(1-\pi_N)\int (K'_S(z,k;\mathbf{S}) - (1-\delta)k)\, d\mu_S(z,k) \\
&-(1-\pi_N)\int \sum_{i=1,2}(K'_{i,L}(z_1,z_2,k_1,k_2;\mathbf{S}) - (1-\delta)k_i) d\mu_L(z_1,z_2,k_1,k_2) \\
&+\pi_N\Big[\int (1-\delta)k\, d\mu_S(z,k) + \sum_{i=1,2}(1-\delta)k_i\, d\mu_L(z_1,z_2,k_1,k_2)\Big] \\
&-\pi_N\Big[\int_{\gamma>\gamma(\hat{\mathbf{S}})}\int K^E_S(z,0;\mathbf{S}) dG_1(z) dF(\gamma) \\
&+\int_{\gamma<\gamma(\hat{\mathbf{S}})}\int \sum_{i=1,2} K^E_{i,L}(z_1,z_2,0,0;\mathbf{S}) dG_2(z_1,z_2) dF(\gamma)\Big],
\end{aligned}
$$

$$(ii)\ s'(z,k;\mathbf{S}) = 1,\ \forall\ (z,k) \quad s'(z_1,z_2,k_1,k_2;\mathbf{S}) = 1\ \forall\ (z_1,z_2,k_1,k_2),$$

$$
\begin{aligned}
(iii)\ N(S) = & \int N_S^d(z,k;\mathbf{S})d\mu_S(z,k) \\
& + \int \left[\sum_{i=1,2} N_{i,L}^d(z_1,z_2,k_1,k_2;\mathbf{S}) \right] d\mu_L(z_1,z_2,k_1,k_2) \\
& + \int Adj\ Cost_S(z,k;\mathbf{S})d\mu_S(z,k) \\
& + \int Adj\ Cost_L(z_1,z_2,k_1,k_2;\mathbf{S})d\mu_L(z_1,z_2,k_1,k_2) + \int_{\gamma<\hat{\gamma}(\mathbf{S})} \gamma dF(\gamma).
\end{aligned}
$$

Lastly, the evolution of the joint distributions over (k,z) and (k_1,k_2,z_1,z_2) are consistent. That is,

$$\mu_S' = \Gamma_S(\mathbf{S}) \quad \text{and} \quad \mu_L' = \Gamma_L(\mathbf{S})$$

are generated by the policy functions $\{K_S', K_{1,L}', K_{2,L}', K_S^E, K_{1,L}^E, K_{2,L}^E\}$ and the exogenous stochastic evolution of $\{A, z, (z_1,z_2), \sigma_A, \sigma_z\}$.

5 Calibration

5.0.1 Fixed parameters

Table 6 shows the parameters taken from the literature. The model period is a quarter, which corresponds to the empirical analysis. The discount factor is $\beta = 0.99$, the depreciation rate is $\delta = 0.03$ and the labor disutility parameter is $\theta = 2$, which are standard in the literature. Following Winberry (2020), we set the capital share $\alpha = 0.21$ and the labor share $\nu = 0.64$, which implies the total return as 85%. Following Koby and Wolf (2020), we assume 6.5 $\%$ of annual exit rates so that 1.625 $\%$ of firms exit at a quarterly frequency. For the parameters regarding exogenous shocks on productivities and uncertainty, we borrow from Bloom et al. (2018) who estimate the shock processes using establishment-level data. Because the uncertainty shocks are modeled as an increase in the standard deviation of unit-level productivity shocks, their measure of shock processes is the most suitable for this analysis. Following their approach, we assume that a single underlying process σ governs the evolution of both micro σ_z and macro σ_A uncertainties so that

$$\sigma = L \ \Rightarrow\ \sigma_A = \sigma_{A,L} \text{ and } \sigma_z = \sigma_{z,L}, \quad \sigma = H \ \Rightarrow\ \sigma_A = \sigma_{A,H} \text{ and } \sigma_z = \sigma_{z,H}$$

and σ follows a two-state Markov process with the following transition probability:

$$\Pi = \begin{bmatrix} \pi_{L,L} & \pi_{L,H} \\ \pi_{H,L} & \pi_{H,H} \end{bmatrix} \quad \text{where} \quad \pi_{L,L} + \pi_{L,H} = \pi_{H,L} + \pi_{H,H} = 1.$$

Finally, we assume that the marginal cost component of financing cost $\psi_2(\mathbf{S})$ is an increasing function of σ as in Alfaro, Bloom and Lin (2018). Specifically,

$$\psi_2(\sigma) = \begin{cases} \psi_2 & \text{if} \quad \sigma = L \\ 1.38 \times \psi_2 & \text{if} \quad \sigma = H \end{cases} \tag{10}$$

implying the finance cost under high uncertainty is 1.38 times higher than that under low uncertainty.[11]

Table 6: List of fixed parameters

Parameter	Description	Value	Data Source
β	Time discount factor	0.99	Standard
θ	Labor Disutility	2	Standard
δ	Depreciation rate (Physical Capital)	0.03	Standard
α	Capital Share	0.21	Winberry (2020)
ν	Labor Share	0.64	Winberry (2020))
π_N	Measure of New Firms (Exit prob)	0.01625	Koby and Wolf (2020)
ρ_z	AR coeff of z	0.95	Bloom et al. (2018)
ρ_A	AR coeff of A	0.95	Bloom et al. (2018)
$\sigma_{z,L}$	STD of z (Low)	0.051	Bloom et al. (2018)
$\sigma_{A,L}$	STD of A (Low)	0.0067	Bloom et al. (2018)
$\sigma_{z,H}$	STD of z (High)	4.1 $\times \sigma_{z,L}$	Bloom et al. (2018)
$\sigma_{A,H}$	STD of A (High)	1.6 $\times \sigma_{A,L}$	Bloom et al. (2018)
$\pi_{L,H}$	Transition prob from Low to High	0.026	Bloom et al. (2018)
$\pi_{H,H}$	Transition prob from High to High	0.943	Bloom et al. (2018)
$\psi_2(H)$	Marginal finance cost (High)	1.38 $\times \psi_2$	Alfaro, Bloom and Lin (2018)

5.0.2 Fitted parameters

The key parameters in the model are the adjustment costs. In particular, the degree to which the investment of a multi-unit firm is different from that of a single-unit firm crucially depends on the relative magnitude between the physical capital adjustment cost and finan-

[11] ψ_2 is the marginal finance cost under low uncertainty and is the fitted parameter.

cial adjustment costs. If physical capital adjustment costs outweigh financial costs, unit-level friction dominates a firm's investment behavior so that single- and multi-unit firms will show little difference. On the other hand, the high value of finance costs makes the boundary of the firm more important so that the investment behavior of multi-unit firms will be significantly different from that of single-unit firms. Therefore, precisely estimating the adjustment costs places a discipline on the degree of difference between single- and multi-unit firms' investment behavior. The correlation between units within a multi-unit firm is also important to determine the gap between single- and multi-unit firms because more diversified multi-unit firms are less responsive to uncertainty shocks. Therefore, we calibrate all fixed-adjustment costs regarding unit-level and firm-level frictions and the correlation across units within a multi-unit firm by matching the following cross-sectional empirical moments in the next section.[12]

Table 7: List of fitted parameters

Parameter	**Description**	**Value**
$\bar{\gamma}$	Upper Bound for Distribution of γ	4.03
$\sigma_{1,2}$	Corr bw z_1 and z_2 within a firm	0.2845
ϕ_F	Fixed adjustment cost (Physical Capital)	0.042
ϕ_C	Convex adjustment cost (Physical Capital)	0.217
ψ_1	Finance cost (Fixed)	0.001
ψ_2	Finance cost (Proportion)	0.18

5.0.3 Targeted moments

We mainly target moments of establishment-level investment rates in Census micro data reported by Cooper and Haltiwanger (2006) and Kehrig and Vincent (2019) Table 8 shows the targeted moments. None of the values are taken from Compustat – but in the following section, we compare the moments from the model to the results from the empirical analysis. The first four moments in Table 8 are known to be informative for adjustment costs in the literature (Cooper and Haltiwanger (2006) and Winberry (2020)). As illustrated by Koby

[12]Consistent with the data moments, we consider only surviving firms in the model. To calculate the cross-sectional moments, we shut down all aggregate shocks, derive a stationary equilibrium, simulate 5000 firms and collect the simulated data at the production-unit level. Since all targeted moments are annual frequency, we aggregate the data from the model up to the yearly horizon when we calculate the moments.

and Wolf (2020), whether firm-level heterogeneity has important implications for aggregate investment dynamics crucially depends on the degree of semi-elasticity of investment to the interest rate. Large elasticity implies that a firm's investment is highly sensitive to interest rate changes so that the general equilibrium smoothing effect from real interest rate adjustment will be strong enough to offset the initial large drop of a single-unit firm's investment. In that case, firm-level heterogeneity will have limited implications for aggregate investment dynamics. Hence, we target semi-elasticity to precisely evaluate the importance of firm-level heterogeneity for aggregate investment.

Table 8: Targeted moments (annual)

Target	**Data**	**Model**
Average Investment Rate (%)	12.2%	16.1%
Standard Deviation of Investment Rates (%)	33.7%	37.5 %
Spike Rate (%)	18.6%	19.0 %
Serial Correlation of Investment Rate	0.058	0.14
Elasticity to real interest rate shock	5.0	5.34
Variance Share of i/k within Multi-Unit Firms (%)	66.3%	62.0 %
Multi-Unit Firm's Output Share (%)	78%	77.3 %

To precisely calibrate the relative magnitude between physical investment adjustment costs and financial adjustment costs, we target the share of variance in the investment rate accounted for by variation within a multi-unit firm. Kehrig and Vincent (2019) find that among the total variance of establishment-level investment rate, 66.3% is explained by the within-firm variation. They show that a large variance in the investment rate within a multi-unit firm arises when firms tend to invest large amounts into a few production units. This pattern of a multi-unit firm's staggered investment largely depends on the relative magnitude of investment adjustment cost and finance cost. If the investment adjustment cost is too high, whenever a firm decides to make a new investment, it would need extra funds. In this case, the incentive to focus the investment on one unit would be weaker. On the other hand, if the financial friction is severe and the investment adjustment cost is not too high, a firm would want to invest in one particular unit at a given time. Therefore, we target the variance share within a multi-unit firm to precisely estimate the adjustment costs.[13] Finally, the

[13]See appendix for how to calculate the variance share.

multi-unit firm's output share is informative for the upper bound of random fixed cost $\bar{\gamma}$ to be a multi-unit firm upon entry.

5.0.4 Calibration results

Table 7 lists the fitted parameters and the calibrated results. The correlation across units within a multi-firm is calibrated as 0.2845, which implies that approximately one-third of unit-level productivity variance arises from firm-level shocks. The fixed adjustment costs of physical capital (0.042) and finance costs (0.18) are broadly consistent with the existing study by Alfaro, Bloom and Lin (2018), who estimate both fixed adjustment costs (0.036) and proportional finance costs (0.1435) using firm-level data. The convex adjustment cost is lower than that in existing studies, e.g., 0.7 in Koby and Wolf (2020), because the finance costs have a smoothing effect on the multi-unit firm's investment.

Table 9: Results for empirical data vs. model data

Dependent variable: $\Delta logk_{j,t+1}$

	Empirical			Model		
	1	2	3	4	5	6
Size $\times$ Uncertainty	0.29**		0.13	0.11		0.044
	(0.13)		(0.12)			
Number of units $\times$ Uncertainty		0.28***	0.25***		0.13	0.11
		(0.07)	(0.06)			
R^2	0.1126	0.1128	0.1128	0.1830	0.1828	0.1834

Note: Standard errors in parentheses are two-way clustered by firm and time.

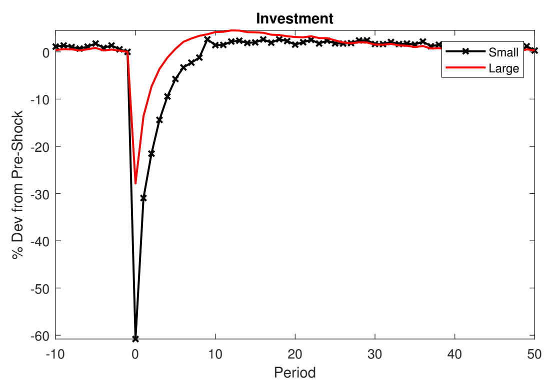
5.0.5 Model-induced regression results

We compare the model with the empirical findings by running the same regression with the model-generated firm panel, which is simulated after we solve the general equilibrium with the calibrated parameters. We consider the fact that Compustat contains only publicly listed firms, whose median time of IPO is approximately 7 years (Wilmer and Pickering (2017)). Hence, we select firms that have survived for at least 7 years after being born. Table 9 shows the results. The coefficient estimate of the size and uncertainty interaction is 0.11 and that of the number of production units and uncertainty interaction is 0.13, which are roughly less than half of the data counterparts. Furthermore, once we control both interaction terms, the size effect decreases by more than half in magnitude, but the coefficient estimate of the number of production units is not as affected as the size effect, which is consistent with the empirical patterns.[14]

6 Inspecting the mechanism

This section explores the underlying mechanism of firm-level decisions before proceeding to general equilibrium analysis. To precisely understand how uncertainty shocks affect a firm's investment choice, we fix all prices at the stationary equilibrium level and investigate only the incumbent firm's investment responses.[15] All parameter values are based on the previous section. We first provide different versions of the impulse responses and then investigate a firm's investment policy function.

Figure 1: Impulse response functions under the partial equilibrium (1)



6.0.1 Impulse responses

Figure 1 plots the impulse responses of investment to uncertainty shocks among single and multi-unit firms, given the prices. To calculate the impulse responses, we simulate 2000 independent economies with 100 quarters. Starting from the stationary equilibrium without

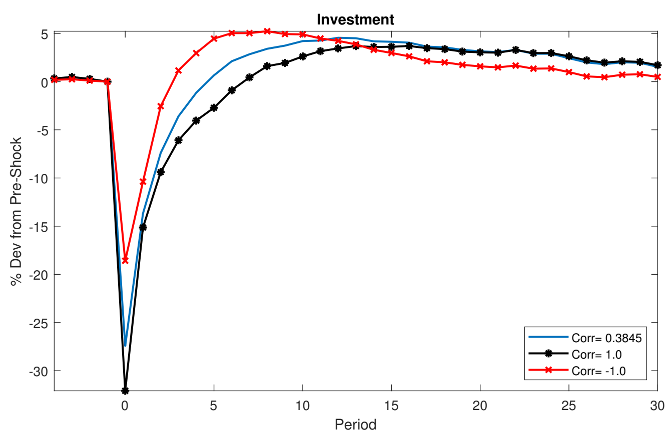
[14]We do not report standard errors for model-generated data because the number of obervation in the simulation is not comparable to the data counterpart.

[15]To calculate the stationary equilibrium, we shut down all the aggregate exogenous shocks, i.e. A = 1, $\sigma = \sigma_L$.

any aggregate shocks, all exogenous shocks to aggregate TFP and uncertainty evolve normally according to the stochastic processes described in the previous sections before period 45. At period 45, we artificially impose a high level of uncertainty. After the shock period, the exogenous processes evolve normally again from period 46.[16] Upon the shock, all types of firms reduce their investments but to different extents, which successfully captures the differential impact of uncertainty shocks. Specifically, the single-unit firms reduce their investment by 61 $\%$, which is approximately more than twice as large as the multi-unit firm's response of 29 $\%$.

A natural way to explain this finding is the diversification benefit of multi-unit firms because idiosyncratic shocks are production-unit-specific and multi-unit firms can diversify the shocks as long as they are not perfectly correlated. However, the diversification proved not to be the sole factor driving heterogeneous responses. Figure 2 compares several investment responses of multi-unit firms with different levels of correlation across units. Clearly, as the correlation becomes negative, i.e., the shocks are more diversified, the impact of uncertainty shocks is more alleviated. However, even in the case of perfect correlation, which is the case of no-diversification benefit, the response of a multi-unit firm's investment is 32 $\%$, which is still more muted than that of a single unit firm's response of 61 $\%$. This result implies that a significant factor other than diversification distinguishes the multi-unit firm's investment response.

Figure 2: Impulse response functions under the partial equilibrium (2)

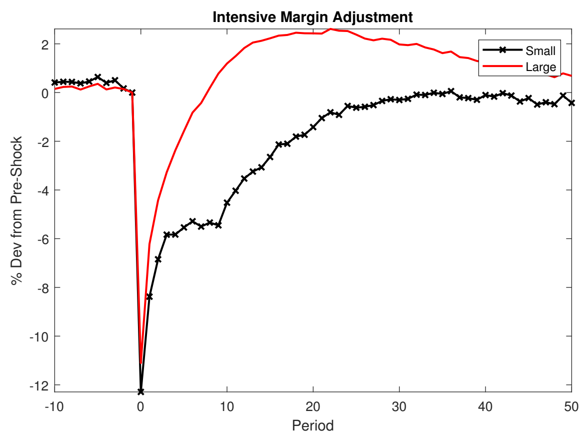


Note: This graph shows impulse response functions of multi-unit firms under different correlation structure.

[16]In the graph, period 0 corresponds to the shock period of 45.

To better understand the mechanism, we decompose the investment response into intensive and extensive margins and identify which margin plays an important role in explaining the differences. Figure 3 shows the investment response due to the intensive margin adjustment. We calculate the investment responses conditional on firms adjusting capital stocks, which is normalized by the measure of adjusting firms.[17] Once we consider only intensive

Figure 3: Intensive vs. Extensive margin adjustments

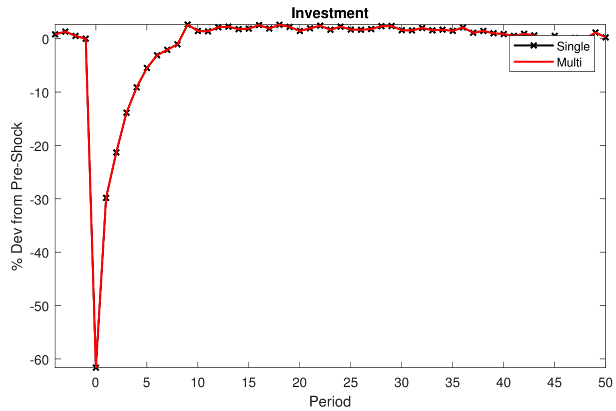


margin adjustment, the gap between single and multi-unit firms becomes smaller than the gap of total investment responses. Specifically, the investment of single-unit firms decreases by 12$\%$ on impact, and that of multi-unit firms declines by 11 $\%$ due to the intensive margin, which shows little difference. This result implies that the gap between single and multi-unit firm responses is driven mainly by the differential sensitivity of extensive margins to uncertainty shocks.

What feature of a multi-unit firm makes the extensive margin less responsive? Unlike a single-unit firm, a multi-unit firm is able to engage in within-firm resource allocation. That is, if a firm does not have enough profit generated by unit 1 to cover expenditure on investment in unit 1, the firm can use the profit from unit 2 without relying on the external finance market. Since utilizing internal capital markets is not available to the firms operating in a single production unit, it is one of the important characteristics of multi-unit firms. In Figure 4, we provide evidence that within-firm resource allocation is the key factor that distinguishes the differential response of single- and multi-unit firms. Figure 4 shows the impulse responses similar to the ones in Figure 1. The only difference from Figure 1 is that multi-unit firms are not allowed to pool the cash flows from two different units without any

[17] See appendix for the derivation of total investment decomposition.

Figure 4: Impulse response functions under the partial equilibrium (3)



Note: For the multi-unit firms, we shut down the internal resource allocation mechanism by assuming that the finance cost is unit-specific rather than firm-specific.

costs. That is, rather than assuming the firm-level financial friction, we assume that financial friction is unit-specific when we calculate the impulse response of multi-unit firms in Figure 4.[18] We further assume that the correlation between shocks to different units is -1, i.e., shocks are perfectly diversified. As we can see in Figure 4, the single- and multi-unit firms' responses are exactly the same despite the perfect diversification.[19] That is, once we eliminate the within-firm allocation mechanism, the difference between single- and multi-unit firms completely disappears. This result illustrates that the dampened investment response of a multi-unit firm crucially relies on the firm's ability to utilize internal capital markets. The following analysis examines the detailed mechanism by investigating each firm's investment policy function.

6.0.2 Policy functions

Figure 5 illustrates the investment policy functions of single- and multi-unit firms under the median level of aggregate TFP and low uncertainty. To be specific, we compare the total investment of two single-unit firms and the total investment of one multi-unit firm under the same states. To simplify the analysis, we consider only the symmetric cases: two single-unit firms that have the same level of capital stock k and productivity z, and one multi-unit

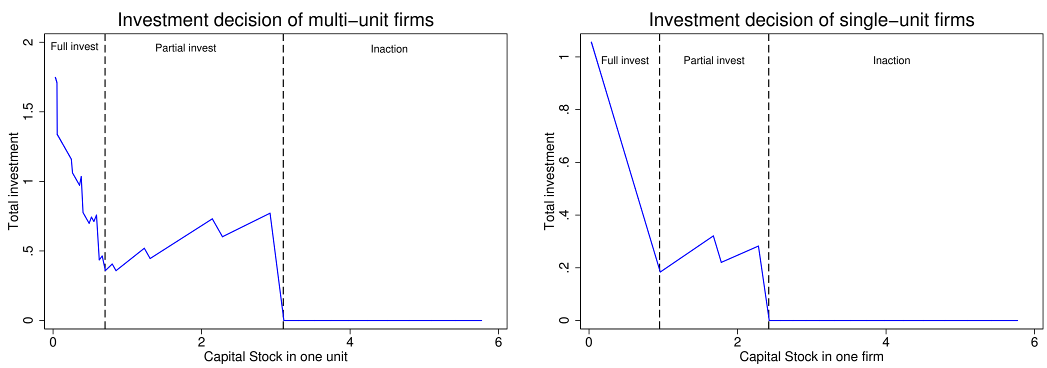
[18]See appendix for the problem of multi-unit firms without internal capital market.

[19]We try different levels of correlation (zero and perfect correlation), but the results are exactly the same. Since all firms are owned by a single representative household, whether an individual firm is diversified or not is not important from the household's perspective.

firm that has the same level of capital stock k and productivity z in each unit. Hence, the bottom panel in Figure 5 plots the sum of investment by two single-unit firms against initial capital stock k in one firm, and the top panel plots the total investment of a multi-unit firm against initial capital stock k in one unit. We fix the idiosyncratic productivities at the highest level and focus on the firm's positive investment decision in this analysis.[20] Furthermore, in order to investigate the pure 'wait-and-see' effect and rule out the diversification benefit, we assume zero convex capital adjustment cost and the perfect correlation between units within a multi-unit firm.

As we can see in Figure 5, both single- and multi-unit firms show qualitatively similar

Figure 5: Policy functions under low uncertainty



Note: Total investment of one multi-unit firm (top) and two single-unit firms (bottom) under low uncertainty. We only consider the symmetric case where a multi-unit firm has the same level of capital stock in each unit $k_1 = k_2 = k$ and two single-unit firms have the same level of capital stock k. Horizontal axis in top panel represents the level of capital stock in one unit, and that in bottom panel represents the level of capital stock in one firm. We fix the idiosyncratic productivity at the highest value and the aggregate TFP at the median level.

investment patterns. There is one region with zero investment (inaction region) and the other region with positive investment (investment region). The investment region is further decomposed into two distinct parts – one in which firms do not rely on external finance (partial investment region) and the other in which firms raise new equity for their expenditure on investment (full investment region). In the partial investment region, the presence of financial friction prevents firms from investing the full amount because the cost of using

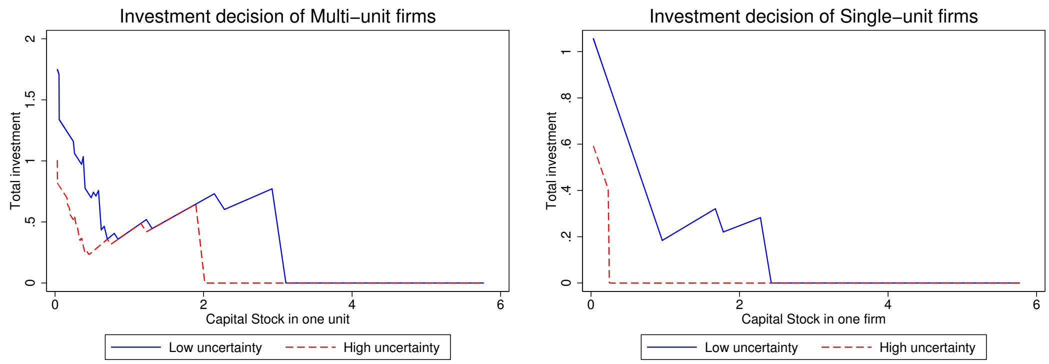
[20]This analysis does not cover disinvestment decisions because (i) single- and multi-unit firms do not show any differences in disinvestment behavior, and (ii) a measure of firms who indeed reduce their investment is very small in my calibrated model due to the depreciation of physical capital.

external finance is greater than the benefit. Hence, the firms optimally choose the constrained amount of investment that can be financed by internal funds.

Despite the qualitative similarity, the policy functions of single- and multi-unit firms show quantitative differences mainly due to the multi-unit firm's ability to engage in internal resource reallocation. First, the investment region of multi-unit firms is wider than that of single-unit firms. The situation arises when there is a good investment opportunity in one particular unit, but expenditure on the investment cannot be financed by the sole cash flow from the same unit. In this case, a multi-unit firm can use the profit from the other unit to cover the expenditure, but a single-unit firm would give up the opportunity because it does not have such an option and the investment opportunity is insufficient to compensate for costly external finance. Second, single-unit firms rely more than multi-unit firms on external finance, i.e., the full investment region of a single-unit firm is wider than that of a multi-unit firm. One interesting finding that is not seen in Figure 5 is that under the partial investment region, a multi-unit firm invests in one particular unit rather than in both units. This result is caused by the two different frictions – (i) unit-level investment fixed cost and (ii) firm-level finance cost. In the presence of finance costs, a multi-unit firm tends to utilize internal funds for investment and tries to avoid allocating a large amount of new capital to both units within the same period. At the same time, due to the fixed investment adjustment cost, the small amount of investment is not profitable. As a result, the firm gives up investing simultaneously in both units and focuses its investment on one particular unit even though both units give exactly the same investment return. This pattern implies that the firm's investment decision for one unit crucially depends on the investment choice of the other unit, i.e., there is inter-dependence of investment within a multi-unit firm.

Figure 6 shows an exercise to investigate the effect of an increase in uncertainty. As in Figure 5, solid blue lines denote the policy functions under low uncertainty, and red dashed lines denote the policy functions under high uncertainty. As the top and bottom panels show, an increase in uncertainty enlarges the inaction regions of all types of firms through a 'wait-and-see' effect, but the effect is especially weaker for multi-unit firms. Similar to single-unit firms, multi-unit firms want to pause their investment when uncertainty is higher. However, for a multi-unit firm, the decision to postpone one particular investment project enlarges internal funds and so helps to relax the constraint on the amount a firm can invest in the other unit. Therefore, rather than delaying all the investment projects within its boundary, a multi-unit firm optimally chooses to delay one of its investment opportunities due to the 'wait-and-see' effect, but at the same time, the firm chooses to keep positive investment in

Figure 6: Policy functions under high uncertainty

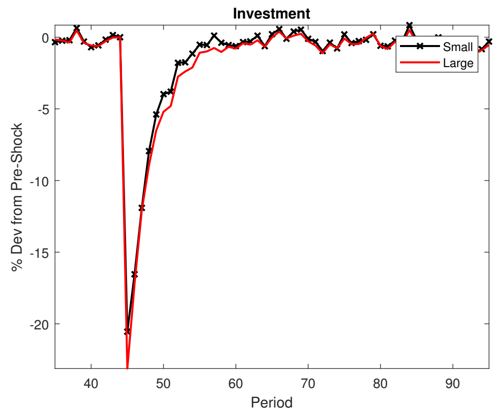


Note: Total investment of one multi-unit firm (top) and two single-unit firms (bottom) under low uncertainty (blue solid line) and high uncertainty (red dashed line). We only consider the symmetric case where a multi-unit firm has the same level of capital stock in each unit $k_1 = k_2 = k$ and two single-unit firms have the same level of capital stock k. Horizontal axis in top panel represents the level of capital stock in one unit, and that in bottom panel represents the level of capital stock in one firm. We fix the idiosyncratic productivity at the highest value and the aggregate TFP at the median level.

the other unit because there are more internal funds available. That is, the multi-unit firm's dampened response arises mainly from the inter-dependence of investment within a firm. The top panel in Figure 6 illustrates the mechanism – a significant portion of multi-unit firms that planned to invest in both units (those in full investment region) under low uncertainty decide to invest in one unit under high uncertainty. Clearly, this mechanism is unavailable to a single-unit firm because it has one investment opportunity in its boundary. Hence, single-unit firms have no choice but to delay their investment. This prediction is confirmed in the bottom panel in Figure 6 – most single-unit firms that planned to invest (especially those in full investment region) under low uncertainty decide not to invest under high uncertainty. Therefore, the response of single-unit firms is larger than that of multi-unit firms.

The different investment responses between single- and multi-unit firms crucially rely on the nature of shocks. Since the increase in uncertainty is modeled as a rise in the variance of next period productivities, a multi-unit firm decides to postpone its investment choice in both units initially without having a direct effect on the resources (profits) available to the firm. Therefore, the initial investment freeze in each unit could have offsetting effects because there are still enough funds available for the firm's investment project. In this regard, the first moment shock, i.e., a negative TFP shock, would have different implications because it directly affects the funds available to the firm. Figure 7 plots the impulse responses of

Figure 7: Impulse response functions to negative aggregate TFP shock (-2% from the median value)



investment to the negative TFP shock among single and multi-unit firms given the prices. In response to this shock, a multi-unit firm reduces its investment slightly more than a single-unit firm does. Therefore, the mechanism explaining the dampened effect of uncertainty shocks does not work and has the opposite prediction in the case of adverse TFP shocks.

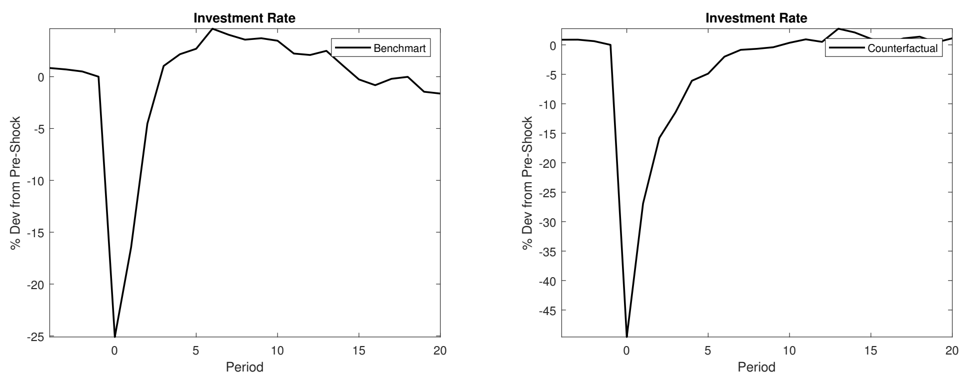
6.0.3 Supportive evidence for the mechanism in the literature

The main mechanism that distinguishes a multi-unit firm's investment behavior is driven by the negative interdependence of investment within a firm. The negative relationship would be more pronounced if firm-level financial friction were more severe because it arises from real and financial frictions. Kehrig and Vincent (2019) provide empirical evidence that the negative investment relationship within a multi-unit firm is indeed stronger in financially constrained firms, i.e., if multi-unit firms are more financially constrained, they tend to rotate the investment across plants rather than invest in both plants. In terms of a firm's alternating investment behavior, Becker et al. (2006) show that the fraction of zero investment and that of investment spikes are significantly lower at the firm level than at the plant level. They argue that those patterns suggest that firms smooth their investment but also concentrate on specific plants within a firm.

7 Aggregate implications

This section explores the aggregate implications of the heterogeneity in firms' investment decisions. Specifically, we examine how much the multi-unit firm's dampened responses contribute to alleviating the impact of uncertainty shocks on aggregate investment responses. To answer this question, we compare the aggregate investment response of the benchmark economy to that of a counterfactual economy with only single-unit firms. Both economies share the same parameters calibrated in the previous section. Since the infinite-dimensional distributions are state variables of individual firms, our model solution heavily relies on the numerical method by Krusell and Smith (1998). Details on the computation method is available in the appendix.

Figure 8: Impulse response functions under the general equilibrium (1)



Note: This graph shows IRFs of investment to uncertainty shocks from the benchmark economy (left) and the counterfactual economy (right).

After we find the equilibrium, we calculate the impulse responses by simulating 2000 independent economies with 100 quarters. Each economy starts with the low uncertainty and median value of aggregate TFP. All exogenous processes evolve normally before period 45. At period 45, we artificially increase the level of uncertainty. After the shock period, the exogenous processes evolve normally again from period 46.[21]

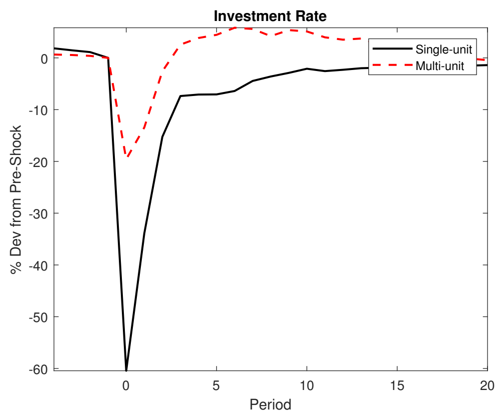
Figure 8 shows the response of aggregate investment to uncertainty shocks in the benchmark economy in the left panel, and the right panel shows the response in the counterfactual panel. In response to uncertainty shocks, investments in both economies decline, but the magnitude of the effect is much smaller for the benchmark economy. Furthermore, the

[21] In the graph, period 0 corresponds to the shock period 45.

benchmark economy shows quicker recovery than the counterpart economy, which is mainly because of multi-unit firms. Overall, the dampened response of multi-unit firms helps to mitigate the impact of uncertainty shocks significantly.

Figure 9 shows the responses of single- and multi-unit firm investment upon uncertainty shocks to the benchmark economy. Similar to the partial equilibrium responses, we find that the impact of uncertainty shocks is asymmetric across firms. Multi-unit firms reduce their investment by approximately 20 $\%$, but single-unit firms reduce it by 60$\%$, a response three times greater than the former.

Figure 9: Impulse response functions under the general equilibrium (2)



Note: This graph shows IRFs of investment to uncertainty shocks among single- vs. multi-unit firms under benchmark economy.

An interesting finding is that in our benchmark economy, single-unit firms reduce their investment by 60$\%$ but in the counterfactual economy, single-unit firms reduce by 50$\%$. That is, even though the aggregate investment responds less to the uncertainty, the single-unit firms indeed reduce their investment more in the benchmark economy. This result arises from the fact that, in the counterfactual economy, the general equilibrium smoothing effect is stronger. Since the initial decrease in investment is larger than that in the benchmark economy, the consumption tomorrow will decrease more, which leads to a larger decrease in the real interest rate. Since the real interest rate is the market price representing the cost of the investment, the price adjustment is more favorable to single-unit firms in the counterfactual economy such that the response of the single-unit firm itself is smaller. This result illustrates that the presence of a multi-unit firm makes the single-unit firm's response even worse. However, since the multi-unit firms account for a significant portion of the aggregate output and investment (78$\%$ of output in our benchmark economy) and the gap between single- and multi-unit firms' responses is sizable, the benefit from multi-unit firms is dominant. This finding suggests that the implication of firm-level heterogeneity on the aggregate investment response crucially relies on the general equilibrium adjustment effect and the distribution of firms.

8 Conclusion

In this paper, we show the asymmetric effect of uncertainty shocks on the investment of small and large firms. We argue that the observed size effect arises from the fact that large firms operate in multiple production units but small firms operate in a single unit. This argument is based on two components. First, we empirically show that the observed size effect is explained mostly by the number of business units of a firm. Second, we employ a heterogeneous firm model to account for the empirical results. In the presence of unit-level real and firm-level financial frictions, a multi-unit firm shows the negative interdependence of investment within a firm, which dampens the real options effect due to uncertainty shocks. We find that in equilibrium, the presence of multi-unit firms has an adverse effect on a single-unit firm's investment response. However, the dampened effect of uncertainty shocks to multi-unit firms still has important implications for aggregate investment responses because (i) multi-unit firms account for a significant portion of aggregate investment and (ii) the gap between single- and multi-unit firms' responses is sizable under general equilibrium.

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A Appendix

A.1 Data cleaning

In the main empirical analysis, we merge Compustat Fundamentals Quarterly data with Compustat Segment and CRSP. Sample period is from 1987Q1 to 2017Q4. Data cleaning and construction process closely follow Ottonello and Winberry (2020), Bharath and Shumway (2008) and Decker, D’Erasmo and Moscoso Boedo (2016).

First, we exclude firm observations with following properties.

- Firms are in finance, insurance, and real estate sectors ($sic \in [60, 67]$) and public administration ($sic \in [91, 97]$)

- Firms are not incorporated in the U.S.
- Firms with observation less than 40.

Firm-quarter observations with following properties are also excluded.

- Investment rate belonging the top and bottom 0.5 percent of the distribution.
- Leverage higher than 10.
- Liquidity belonging the top and bottom 0.5 percent of the distribution.
- Current asset to total asset ratio higher than 10 or below -10.
- Quarterly real sales growth rate higher than 1 or below -1.
- Acquisitions larger than 5% of total assets.
- Missing and non-positive value of capital stock (**ppentq**), sales (**salesq**), or total assets (**atq**).
- Missing leverage.

The investment is measured as $\Delta ln(k_{i,t+1}) = ln(k_{i,t+1}) - ln(k_{i,t})$, where $k_{i,t+1}$ is the real book value of capital stock of firm i at the end of period t. To construct the measure of capital stock, the perpetual inventory method is used. Specifically, for each firm, the initial value of capital stock is set by the first reported value of gross plant, property, and equipment (**ppegtq**) of each firm. Then, the series of $k_{i,t+1}$ is computed recursively using the changes of net plant, property, and equipment (**ppentq**):

$$k_{i,t+1} = k_{i,t} + ppentq_t - ppentq_{t-1}$$

Before the process, capital stock is deflated by the non-residential fixed investment good deflator taken from NIPA table. For the missing value of capital stock (**ppenqt**), we impute as follows. If firm's ppentq is missing but the values right before and after the missing value are nonmissing, it is estimated by linear interpolation. If two or more consecutive observation are missing, there is no imputation.

As a proxy of the firm size, the log of sales (**saleq**) or the log of total asset (**atq**) are used. Both are deflated by Implicit Price Deflators for Gross Domestic Product from NIPA table. Another set of variables capturing the firm-level characteristic consists of the leverage, the liquidity, sales growth rate, current asset (**atcq**) to total asset ratio and the number of markets a firm access to. Leverage is the debt to total asset ratio, where the debt is the sum of short-term (**dlcq**) and long-term (**dlttq**) debt. Liquidity is defined as the ratio of cash and short-term investment (**cheq**) to total asset. Sales growth rate is measured as the log-differences in real sales. In order to calculate the number of lines of busienss (the number of different industries firms are operating in), we use Compustat Segment. Since this variable is available in Compustat Segment which only provides an annual frequency data, in a given year, quarterly values are filled-in by using the corresponding yearly value. To construct a distance to default measure, we closely follow Bharath and Shumway (2008). First, we defind the firm-level variable distance to default (dd) as follows

$$dd \equiv \frac{ln(V/F) + (\mu - \sigma^2/2)}{\sigma}$$

where V is the total value of the firm, μ is the annual expected return on V, σ is the annual volatility of the firm's value, and F is the face value of firm's debt (short-term debt (dlcq) plus one-half of long-term debt (dlttq)). In this procedure, it is crucial to estimate the measure of V and we follow an iterative method by Bharath and Shumway (2008).

Step 1. Set a guess V_j for V. For the initial guess V_0, we use the sum of firm's debt and equity, i.e. $V_0 = E + F$, where E is the firm's stock price times the number of shares (both items are available from CRSP).

Step 2. Estimate the mean μ and variance σ^2 of return of the guessed firm's value $\Delta ln(V_j)$ over $T = 250$-day moving window.

Step 3. Obtain a new estimate of V_{j+1} from the Black-Scholes-Merton option-pricing framework. Specifically, we find V_{j+1} such that

$$E = V_{j+t1}\Phi(\delta_1) - e^{-rT}F\ \Phi(\delta_2)$$

where $\delta_1 \equiv \dfrac{ln(V_{j+1}/F) + (\mu - \sigma^2/2)T}{\sigma^2\sqrt{T}}$, $\delta_2 = \delta_1 - \sigma\sqrt{T}$, and r is the daily one-year constant maturity Treasury-yield.

Step 4. Compare V_j and V_{j+1}. If they are close enough, we are done. Otherwise, we repeat the procedure from step 1 with V_{j+1} as a new guess.

A.1.1 Robustness check

In this section, we perform additional robustness check for the baseline analysis. The first column shows the result from estimating

$$\frac{i_{i,t}}{k_{i,t+1}} = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t}, \tag{11}$$

where $i_{i,t}$ is a capital expenditure and so $\frac{i_{i,t}}{k_{i,t+1}}$ is a different measure of investment rate. The following columns perform additional robustness checks in order to deal with the concerns of different cyclical behavior of small and large firms. Specifically, they show the results from estimating

Table 10: Robustness check

	(1)	(2)	(3)	(4)	(5)
size × uncertainty	0.209***	0.319***	0.295**	0.296**	0.275**
	(0.052)	(0.157)	(0.131)	(0.144)	(0.125)
size × GDP growth		0.010			
		(0.022)			
size × interest			0.027		
			(0.072)		
size × inflation				0.023	
				(0.107)	
size × unemp					0.024
					(0.035)
Observations	235,695	240,724	240,724	240,724	240,724
R^2	0.314	0.113	0.113	0.113	0.113

Note: Column 1 uses different investment measure $\frac{i_{j,t}}{k_{j,t-1}}$ and column 2 - 5 controls for different cyclicalities. Standard errors in parentheses are two-way clustered by firm and time. We standardize the size measure over the entire sample. The sample period is from 1987Q1 to 2017Q4, and all firms in Compustat are used for the analysis except those in finance, insurance, real estate and public administration sectors.

$$\frac{i_{i,t}}{k_{i,t+1}} = \alpha_i + \alpha_{s,t} + \beta size_{i,t-1} \times Unc_t + size_{i,t-1} \times Y_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t}, \tag{12}$$

where Y_ts are real GDP growth, interest rate, inflation rate, and unemployment rates. We control the same variables as in the baseline analysis.

In this section, we perform a robustness check regarding – whether a firm's choice on the number of lines of business is systematically related with uncertainty process. This exercise provides a rational for the model assumption that only new firms can choose to be a single- or multi- unit firm.

Table 11: Uncertainty and the number of lines of business

uncertainty	**-0.084**
	(0.068)
gdp growth	-0.000
	(0.000)
interest rate	0.226***
	(0.062)
inflation rate	-0.001*
	(0.001)
unemployment rate	0.001***
	(0.000)
Observations	301,460
R^2	0.030

Note: This table shows the results from estimating $\Delta log(lob_{j,t+1}) = \alpha_i + \alpha_{s,t} + Unc_t + Y_t + \Gamma' Z_{i,t-1} + \epsilon_{i,t}$ Dependent variable is the log change of the number of lines of business. Standard errors in parentheses are two-way clustered by firm and time. The sample period is from 1987Q1 to 2017Q4, and all firms in Compustat are used for the analysis except those in finance, insurance, real estate and public administration sectors.

A.2 Model computation

In the model section, we solve (i) stationary equilibrium, (ii) impulse responses under partial equilibrium and (iii) impulse responses under general equilibrium. Because we assume representative household's preference as

$$ln\ C - \theta N, \quad \theta > 0$$

the wage w is determined by

$$w = \frac{\theta}{U_C} = \theta C.$$

Hence, in order to solve equilibirum, we need to keep track of either U_C or w. We define $p \equiv U_C$ which is the intertemporal price of consumption goods. As in Bloom et al. (2018), this approach enables us to redefine firm's problem in terms of household's marginal utility, which allows us to solve the model relatively simpler. For instance, we transform following incumbent single-unit firm's problem

$$V^S(z,k;\mathbf{S}) = \max_{d,n,k'} \ d - w(\mathbf{S})\psi(d;\mathbf{S}) + E\Big[m(\mathbf{S},\mathbf{S}')V_0^S(z',k';\mathbf{S}') \mid z;\mathbf{S}\Big]$$

into

$$\tilde{V}^S(z,k;\mathbf{S}) = \max_{d,n,k'} \ p(\mathbf{S})(d - w(\mathbf{S})\psi(d;\mathbf{S})) + \beta E\Big[\tilde{V}_0^S(z',k';\mathbf{S}') \mid z;\mathbf{S}\Big].$$

We solve firm's problem by using value function iteration. We set up the grid following $k_i = k_{i-1}/(1-\delta)$ to ensure the depreciated value of k_i to coincide with k_{i-1} and choose the lower and upper bound of grid points of k large enough to prevent firms to choose the boundary points.

A.3 Staionary equilibirum economy

Step 1. Guess a general equilibrium price $p = U_c$.

Step 2. Given the guess, solve for incumbent single- and multi- unit firm's and entrant single- and multi- unit firm's problem.

Step 3. Obtain time invariant measure of μ_S and μ_L from (i) incumbent and entrant firms' optimal choices and (ii) distribution of idiosyncratic productivies.

Step 4. Compute aggregate variables based on firm-level optimal choices and time invariant measure of μ_S and μ_L. If the implied aggregate consumption $C = Y - I$ is consistent with the guessed value of $p = U_c$, we are done. Otherwise, we update p based on the bi-section method and redo the whole process until it converges.

A.3.1 Impulse response functions under partial equilibrium

We fix the equilibrium price $\bar{p} = \bar{U}_C$ and $\bar{w} = \theta/\bar{p}$ at the steady-state level, which is derived from the previous section. Then, we solve individual firms problem given the prices and exogenous processs of aggregate TFP and uncertainty. For instance, we solve the single-unit firms problem as follows

$$\tilde{V}^S(z,k;A,\sigma) = \max_{d,n,k'} \bar{p}(d - \bar{w}\psi(d;A,\sigma)) + \beta E\left[\tilde{V}_0^S(z',k';A',\sigma') \mid z;A,\sigma\right].$$

We solve the incumbent multi-unit firm's problem and entrant single- and multi- unit firm's problems in a similar way. Then, we simulate 2000 independent economies with 100 quarters. For each economy i, starting from the stationary distribution without any aggregate shocks, we switch on all exogeneous shocks to aggregate TFP and uncertainty, allowing them to evolve normally according to the stochastic processes described in the model section before period 45. At period 45, we artificially impose a high value of uncertainty (or low TFP). After the shock period, the exogenous processes evolve normally again from period 46. In each period, we repeatedly calculate the cross sectional distribution of single- and multi-unit firms and calculate aggregate variable of interest X_{it}. With simulated series of $\{X_{it}\}$, we define the time t response of X to an exogenous shock as

$$\hat{X}_t = 100 \times ln\left(\frac{\bar{X}_t}{\bar{X}_{44}}\right),$$

where $\bar{X}_t$ is the level of cross-economy mean: $\frac{1}{2000}\sum_i X_{it}$.

A.3.2 Impulse response functions under general equilibrium

To calculate impulse response functions under general equilbrium, we need to solve the model with exogenous shocks and endogenous prices. In this case, the infinite-dimensional distributions become state variables of individual firms, which is known to be big challenge to solve the model without approximation. Hence, our model solution heavily relies on the numerical method by Krusell and Smith (1998). In this method, we assume that market price p and aggregate capital stock K follow approximate log-linear rules as;

$$ln(p) = \alpha_p(A,\sigma,\sigma_{-1}) + \beta_p(A,\sigma,\sigma_{-1})ln(K)$$

$$ln(K') = \alpha_K(A,\sigma,\sigma_{-1}) + \beta_K(A,\sigma,\sigma_{-1})ln(K)$$

In order to find the approximate law of motion for price and aggregate capital stock, we solve the model as follows.

Step 1. Guess the approximate law of motions for p and K as

$$ln(p) = \alpha_p^{(j)}(A, \sigma, \sigma_{-1}) + \beta_p^{(j)}(A, \sigma, \sigma_{-1})ln(K)$$

$$ln(K') = \alpha_K^{(j)}(A, \sigma, \sigma_{-1}) + \beta_K^{(j)}(A, \sigma, \sigma_{-1})ln(K)$$

Step 2. Given the guess, solve for individual firm's problem. For instance, a single-unit firm solves

$$\tilde{V}^S(z, k; A, \sigma, \sigma_{-1}, K) = \max_{d,n,k'} p(A, \sigma, \sigma_{-1}, K)(d - w(A, \sigma, \sigma_{-1}, K)\psi(d; A, \sigma, \sigma_{-1}))$$

$$+\beta E\left[\tilde{V}_0^S(z', k', K'; A', \sigma', \sigma) \mid z; A, \sigma, \sigma_{-1}, K\right].$$

Step 3. After solving all firms' problem, simulate the economy by allowing all exogenous shocks to evolve normally. When simulating the economy, we update the distribution of single- and multi-unit firms based on the optimal choices of all firms (both incumbents and entrants firms) and exogenous processs.

Step 4. Based on the simulated aggregate data of $\{p_t^{(j)}\}$ and $\{K_t^{(j)}\}$ from Step 3, update the log-linear maping to get the new coefficient in the approximate law of motion for p and K:

$$\alpha_p^{(j+1)}(A, \sigma, \sigma_{-1}),\ \beta_p^{(j+1)}(A, \sigma, \sigma_{-1}),\ \alpha_K^{(j+1)}(A, \sigma, \sigma_{-1}),\ \beta_K^{(j+1)}(A, \sigma, \sigma_{-1}).$$

Step 5. If the coefficients are close enough to the guessed values, we are done. Otherwise, we update the guess.

Given the converged approximate law of motion for p and K, and firm's optimal decisions, we calculate the impulse response functions as in the previous section.

A.4 Additional model analysis

A.4.1 Variance share in Calibration

We calculate the variance share within a firm as follows. The total variance of i/k across production units, denoted by V_T, can be decomposed into two components:

$$\sum_j \omega_j \sum_{n=1}^{N_j} \frac{1}{N_j}\left[\left(\frac{i}{k}\right)_{n,j} - \left(\frac{i}{k}\right)\right]^2 \equiv V_T$$

$$= \underbrace{\sum_j \omega_j \left[\left(\frac{i}{k}\right)_j - \left(\frac{i}{k}\right)\right]^2}_{V_B} + \underbrace{\sum_j \omega_j \sum_{n=1}^{N_j} \frac{1}{N_j}\left[\left(\frac{i}{k}\right)_{n,j} - \left(\frac{i}{k}\right)_j\right]^2}_{V_W}$$

where $\left(\frac{i}{k}\right)_j$ is the mean investment rate within-firm j, $\left(\frac{i}{k}\right)$ is the mean investment rate across all units, N_j is the number of units within firm j, and ω_j is the weight of firm j. The first term V_B is the variance between firms, and the second term V_W represents the average variance across units within a firm. Then, the variance share of i/k within a multi-unit firm is calculated as V_W/V_T.

A.4.2 Investment decomposition - intensive vs. extensive margin

We decompose the total investment responses as follows. For the single-unit firms, the total investment at $t+j$ among them is calculated as

$$I_{t+j} = \int i_{i+j}(z,k)d\mu_{S,t+j}(z,k).$$

Then, the log change of total investment from time t to $t+j$ can be decomposed as

$$ln\ I_{t+j} - ln\ I_t = ln \int i_{i+j}(z,k)d\mu_{S,t+j}(z,k) - ln \int i_i(z,k)d\mu_{S,t}(z,k)$$

$$= ln \int i_{i+j}(z,k)d\mu_{S,t+j}(z,k \mid adjust)\pi_{S,t+j}(adjust)$$

$$-ln \int i_i(z,k)d\mu_{S,t}(z,k \mid adjust)\pi_{S,t}(adjust)$$

$$= \underbrace{ln \int i_{i+j}(z,k) d\mu_{S,t+j}(z,k \mid adjust) - ln \int i_i(z,k) d\mu_{S,t}(z,k \mid adjust)}_{\text{Intensive margin adjustment}}$$

$$+ \underbrace{ln\ \pi_{S,t+j}(adjust) - ln\ \pi_{S,t}(adjust)}_{\text{Extensive margin adjustment}}.$$

where $i_t(z,k)$ is the investment of firm (z,k) at time t, $\mu_{S,t}$ is the distribution of single-unit firms at time t, and $\pi_{S,t}(adjust)$ is the measure of single-unit firms who adjust their investment at time t. For the multi-unit firms, we do this in a similar manner.

A.4.3 Multi-unit firms without internal capital allocation

Without internal capital market, a multi-unit firm solves the following problem.

$$V^L(z_1, z_2, k_1, k_2; \mathbf{S}) = \max_{d_1, d_2, n_1, n_2, k_1', k_2'} d_1 + d_2 - w\psi(d_1; \mathbf{S}) - w\psi(d_2; \mathbf{S})$$

$$+E\bigg[m(\mathbf{S}, \mathbf{S}') V_0^L(z_1', z_2', k_1', k_2'; S') \mid z_1, z_2; \mathbf{S}\bigg]$$

where

$$d_1 = Az_1 k_1^\alpha n_1^\nu - w(\mathbf{S}) n_1 - k_1' + (1-\delta) k_1 - w(\mathbf{S}) \Phi(k_1, k_1')$$

$$d_2 = Az_2 k_2^\alpha n_2^\nu - w(\mathbf{S}) n_2 - k_2' + (1-\delta) k_2 - w(\mathbf{S}) \Phi(k_2, k_2')$$